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1. (25%) Consider the following LP problem.

$$Min\ Z = 5x_1 - 2x_2 + 3x_3$$
$$\text{s.t. } 2x_1 + x_2 + x_3 = 10$$
$$3x_1 - x_2 + 2x_3 \geq 6$$
$$x_1 + 2x_2 \leq 8$$
$$x_1, x_2 \geq 0,\ x_3 \text{ unrestricted}$$

(a) Suppose that the optimal solution is $(x_1, x_2, x_3) = (0, 4, 6)$. Find the basic variables and the corresponding optimal basis when simplex method is applied to solve this problem. (5%)

(b) Write down the dual of the LP problem (using y_1, y_2, y_3 as dual variables). (5%)

(c) Compute the dual optimal solution. (5%)

(d) Find the range of C_3 (currently $C_3 = 3$) for the primal problem such that the current optimal solution $(x_1, x_2, x_3) = (0, 4, 6)$ remains unchanged. (5%)

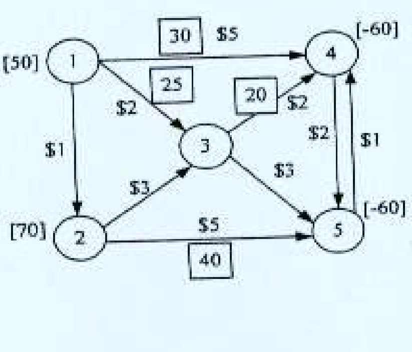
(e) Find the range of b_3 (currently $b_3 = 8$) for the primal problem such that the current dual optimal solution remains unchanged. (5%)

2. (20%) Consider a minimum cost flow problem with the following network: node 1 supplies 50 units, node 2 supplies 70 units, node 3 is a transfer station (no supply and no demand), node 4 demands 60 units, and node 5 requires 60 units. The dollar amount associated with each arc represents unit transportation cost; for example, transportation cost from node 1 to node 4 is 5 dollars per unit. In addition, several arcs have limited transportation capacities. For example, node 1 can transport a maximum of 30 units to node 4.

(a) Apply network simplex method with upper bound technique to examine whether the following solution is optimal: $X_{14} = 30$, $X_{13} = 20$, $X_{23} = 30$, $X_{25} = 40$, $X_{34} = 20$, $X_{35} = 30$, and $X_{54} = 10$. If not, continue to search for an optimal solution. (10%)

(b) What is the range of unit transportation cost of arc 3-5 (currently $C_{35} = 3$) such that the current optimal solution remains unchanged? (5%)

(c) Suppose that node 1 increases its supply quantity by 10 units (from 50 to 60), and node 5 also increases its demand quantity by 10 units (from 60 to 70). What is the new optimal solution? (5%)



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3. (15%) Solve the following problem by dynamic programming technique.

$$Max\ Z = X_1^3 + 5X_2^2 + 30X_3$$
$$s.t.\quad X_1 \cdot X_2 \cdot X_3 = 6$$
$$X_1 \ge 1, X_2 \ge 1, X_3 \ge 1 \text{ and are integers}$$

4. (25%) In an M/M/2/4 system, where the customers arrive according to a Poisson process at a mean rate of 8 per hour, and the service times are i.i.d. exponentially distributed with a mean service rate of 4 per hour. Answer the following questions.

(a) Construct the rate-in rate-out diagram for this problem and compute steady-state probability distribution for the number of customers in the system. (8%)

(b) Calculate the expected waiting time of customers in the system in the steady state. (6%)

(c) Suppose the system will receive 50 dollars per customer served, and will suffer a penalty cost of 20 dollars per hour for each customer in queue. In the long run, what is mean profit per hour for the system? (4%)

(d) Suppose at time t, a customer Tom arrives and sees two customers in the system (both currently being served). What is the mean and standard deviation of Tom's waiting time in the system (including his service time)? (4%)

(e) Suppose the system is full (four customers) now. What is the probability that next arrival will depart immediately from the system (without entering the system)? (3%)

5. (15%) Consider a Markov chain with the following one-step transition probability matrix.

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.3 & 0.4 & 0.3 \\ 0.2 & 0.4 & 0.4 \\ 0.5 & 0.2 & 0.3 \end{bmatrix} \end{array}$$

(1) Compute $\Pr\{X_{20} = 1 \mid X_{18} = 1\}$. (5%)

(2) Compute $\Pr\{X_2 = 2\}$ given that $\Pr\{X_1 = 0\} = 1/5$, $\Pr\{X_1 = 1\} = \Pr\{X_1 = 2\} = 2/5$. (5%)

(3) This Markov chain is doubly stochastic. Compute the steady state probability of state 1. (5%)