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1. 按題目順序作答並且題號標示清楚, 否則不予評分。
2. 每題答案應有合理的步驟說明, 否則不給分。

1. (10 pts.) State the *Bayes' Theorem* in detail.

2. (4+8+8 pts.) A box contains four marbles - one red, one green, one blue and one white. Consider an experiment that consisits of taking one marble from the box, then replacing it in the box and drawing a second marble from the box, again, replacing the second marble in the box and drawing a third one from the box. Let X denote the number of red marbles drawn in the three drawings.

(a) Describe the sample space.

(b) Find the probability mass function of X.

(c) If the drawing new marbles without replacing the previous drawn marbles, find the probability mass function of X.

3. (10 pts.) Consider a set of 23 unrelated people. Because each pair of people shares the same birthday with probability 1/365, and there are $\binom{23}{2} = 253$ pairs, why isn't the probability that at least two people have the same birthday eaqual to 253/365?

4. (10 pts.) Suppose that X and Y are independent random variables having the common density function

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Find the density fuction of the random variable $Z = X/Y$.

5. (15 pts.) Suppose that $X_1, X_2, \cdots, X_n$ are normal with mean μ_1; $Y_1, \cdots, Y_n$ are normal with mean μ_2; and $W_1, \cdots, W_n$ are nomal with mean $\mu_1 + \mu_2$. Assuming that all $3n$ random variables are independent with a common variance, derive the maximum likelihood estimators of μ_1 and μ_2. The probability density function of normal distribution with mean μ and variance σ^2 is $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$, $-\infty < x < \infty$.

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6. (5 pts. each) A new cure has been developed for a certain type of cement that results in a compressive strength of 5000 kilograms per square centimeter and a standard deviation of 120. To test the hypothesis that $\mu = 5000$ against the alternative that $\mu < 5000$, a random sample of 36 pieces of cement is tested. The critical region is defined to be $\bar{x} < 4975$.

(a) Name the sampling distribution $\bar{X}$.

(b) Evaluate the probability of committing a type I error, α, when H_0 is true. Express your answer in terms of $\Phi(a) = P(Z < a)$, where Z follows the standard normal distribution.

(c) Evaluate the probability of committing a type II error, β, for the alternative $\mu = 4960$. Express your answer in terms of $\Phi(a)$.

(d) Is this a good test procedure? Why or why not?

(e) Modify the critical region such that the probability of committing a type I error is 0.05.

(f) Based on the new critical region from (e), shall we increse or reduce the sample size 36 if the power is at least 0.9 for the alternative $\mu = 4960$? Find the new sample size.

(g) Compute the P-value if the average sample strength from 36 pieces of cement is 4965. Make a conclusion at the significance level 0.05.

You might need the following information:
$\Phi(1.28) = 0.9$; $\Phi(1.645) = 0.95$; $\Phi(1.96) = 0.975$.