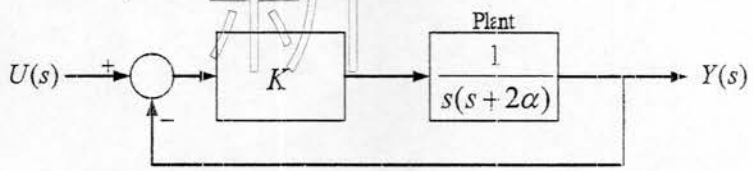
1. (20%) Explain the following terminologies briefly:
   - **(a)** Zero-state response. (5%)
   - **(b)** Resonant frequency. (5%)
   - **(c)** Final value theorem. (5%)
   - **(d)** Linear time-invariant system. (5%)

2. (30%) For a unit-step input $u(t)$ and a given positive constant α, the unity feedback second-order control system is shown as follows:



   - **(a)** Determine the values of control gain K so that the system can be classified to be **(i)** underdamped, **(ii)** overdamped, or **(iii)** critically damped, if possible. (10%)
   - **(b)** Roughly sketch the system output $y(t)$ for each case in part (a). (10%)
   - **(c)** If $\alpha = K$, select the value of K to result in a zero-overshoot system response with the rapidest rise time, and then find this response. (10%)

3. (20%) Consider the series RLC circuit (where the resistance R, inductance L and the capacitance C are constants) in series with the voltage input $v(t)$.
   - **(a)** Let the state variables be $x_1(t) = v_C(t)$, which is the voltage across the capacitor, and $x_2(t) = i_L(t)$, which is the current through the inductor. The voltage $v_R(t)$ across the resistor is the system output. Find both state variable and transfer function models for this system. (10%)
   - **(b)** Determine the conditions for R, L and C to yield an oscillator circuit. Explain the meanings of your answer briefly. (10%)

4. (30%) A 2nd-order system is represented by the differential equation with the input $u(t)$ and the output $y(t)$ as follows:

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = \frac{du}{dt} + 2u\,.$$

   - **(a)** Find the state variable model of this system in controllable canonical form (phase variable format) to determine if it is **(i)** controllable, **(ii)** observable, and **(iii)** stable. Why? (10%)
   - **(b)** Determine whether the system whose state variables are redefined as $x_1 = y$ and $x_2 = \frac{dy}{dt} - u$ is **(i)** controllable, **(ii)** observable, and **(iii)** stable. Why? (10%)
   - **(c)** For a state feedback design with $u(t) = -Kx(t)$, find the gain matrix K to obtain the resulting closed-loop system having the repeated poles at $s = -1$. (10%)