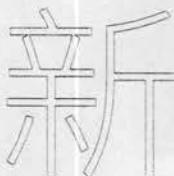


## 科目：工程數學一(線性代數)

編號：353、362 適用：電機系、電機系(通訊工程)

考生注意：

1. 依次序作答，只要標明題號，不必抄題。
2. 答案必須寫在答案卷上，否則不予計分。
3. 限用藍、黑色筆作答；試題須隨卷繳回。

本試題  
共 / 頁  
第 / 頁(一) (24%, 每小題 4%) Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be a linear transformationand that  $T(1, 0) = (2, 7)$ ,  $T(1, 1) = (4, 5)$ (a) What is  $T(3, 5)$ ?(b) What is  $T(a^2, b^2)$ ?(c) If  $T(p, q) = (18, 54)$ , Find  $p, q$ ?(d) Find the null space of  $T$ ?(e) Find the range of  $T$ ?(f) Is  $T$  one-to-one?

(二) (18%, 每小題 6%) Consider the following Linear system equation

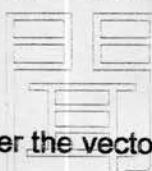
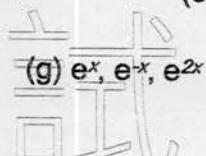
$$\begin{cases} kx + y + z = 1 \\ x + ky + z = 1 \\ x + y + kz = 1 \end{cases}$$

Determine  $k$  condition to make system:

(a) A unique solution

(b) No solution

(c) Infinitely many solutions

(三) (24%, 每小題 3%) Consider the vector space of all functions of a variable  $x$ .Determine the following sets of functions are linear independent or linear dependent. (show details)(a)  $\sin x, \cos x$ (b)  $x, x^2$ (c)  $x, \sin x$ (d)  $\ln(x), \ln(x^4)$ (e)  $0, \tan x$ (f)  $x, x^2, x^3$ (g)  $e^x, e^{-x}, e^{2x}$ (h)  $\sin^2 x, \cos^2 x, \cos 2x$ (四) (34%, (a)~(f)每小題 4%; (g)~(h)每小題 5%) Let  $A = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$  and  $\det(A)=6$ ,

Find:

(a)  $\det(-A)$ (b)  $\det(-2A)$ (c)  $\det(A^{-1})$ (d)  $\det(2A^{-1})$ (e)  $\det(\text{adj}(A))$ (f)  $\det \begin{bmatrix} a+2p & -x & 3p \\ b+2q & -y & 3q \\ c+2r & -z & 3r \end{bmatrix}$ (g) If  $b=c=r=p=x=y=0$ , and  $a > q > z > 0$ , Find  $a, q, z$ .(h) If  $b=c=r=p=x=y=0$ , and  $a > q > z > 0$ , Find eigenvalues of  $A$ .