國立中正大學106學年度碩士班招生考試試題

系所別：經濟學系國際經濟學-乙組　　科目：微積分

第1節　　

請回答下述問題（並詳述推理與計算過程）

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a real-valued function, and let $a, b \in \mathbb{R}$ be two real numbers. Please write down the $\epsilon - \delta$ definition for each following statement:

   (1.1) $f(x)$ has a limit y at $x = a$. **(10 pts.)**

   (1.2) $f(x)$ is continuous at $x = a$. **(10 pts.)**

   (1.3) $f(x)$ is differentiable at $x = a$. **(10 pts.)**

2. Determine if each following series converges or diverges. If it converges determine its sum.

   (2.1) $\sum_{k=2}^{\infty} \frac{1}{k^2-1}$. **(7 pts.)**

   (2.2) $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k}$. **(7 pts.)**

   (2.3) $\sum_{k=0}^{\infty} \frac{k^2-k^3}{k^3+1}$. **(7 pts.)**

3. Find $f'(x)$ for each following function:

   (3.1) $f(x) = \int_{-x}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$, where $x \in \mathbb{R}_+$. **(7 pts.)**

   (3.2) $x^2 + [f(x)]^2 - 5x + 8f(x) + 2x[f(x)]^2 = 5$. **(7 pts.)**

4. Evaluate the following integrals:

   (4.1) $\int_0^{\infty} xe^{-x} dx$. **(7 pts.)**

   (4.2) $\int_0^1 x^3\sqrt{1-x^2} dx$. **(7 pts.)**

   (4.3) $\int_{-\infty}^{\infty} e^{3x} \left(\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \right) dx$. **(7 pts.)**

5. Find the Taylor approximation of order **two** of each following function at the corresponding point:

   (5.1) $f(x) = ln(1 + x^2)$ at $x = 0$. **(7 pts.)**

   (5.2) $F(x, y) = e^{x^2+y}$ at $(x, y) = (1, 1)$. **(7 pts.)**