國立彰化師範大學106學年度碩士班招生考試試題

系所：電子工程學系(乙組選考戊)、資訊工程學系(選考乙)、資訊工程學系積體電路設計碩士班(選考戊)　　科目：離散數學

☆☆請在答案紙上作答☆☆　　

1. Construct the truth table for the following propositions. (10%)
   (a) $p \leftrightarrow q$
   (b) $\neg p \rightarrow (q \rightarrow r)$

2. (a) Prove that $1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + (n-1) \cdot n$ is $O(n^3)$. (7%)
   (b) Give a good big-O estimate for: $n^{2^n} + n^{n^2}$. (3%)

3. Suppose A = $\{a, b, c\}$. Mark the following statement TRUE or FALSE. (8%)
   (a) $\{b, c\} \in P(\text{A})$.
   (b) $\{\{a\}\} \subseteq P(\text{A})$.
   (c) $\emptyset \subseteq \text{A}$.
   (d) $\{\emptyset\} \subseteq P(\text{A})$.
   (e) $\emptyset \subseteq \text{A} \times \text{A}$.
   (f) $\{a, c\} \in \text{A}$.
   (g) $\{a, b\} \in \text{A} \times \text{A}$.
   (h) $(c, c) \in \text{A} \times \text{A}$.

4. Find the solution to each of these recurrence relations and initial conditions. (12%)
   (a) $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$.
   (b) $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 1$, $a_1 = 0$.

5. Prove or disprove: if A, B, and C are sets, then $\text{A} - (\text{B} \cap \text{C}) = (\text{A} - \text{B}) \cup (\text{A} - \text{C})$. (10%)

6. Find all solutions, if any, to the system of congruences $x \equiv 5 \pmod 6$, $x \equiv 3 \pmod{10}$, and $x \equiv 8 \pmod{15}$. (10%)

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7. Prove that if n is a positive integer, then 21 divides $4^{n+1} + 5^{2n-1}$. (10%)

8. If $x_1, x_2, x_3, x_4, x_5, x_6$ are nonnegative integers. How many solutions are there to equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, such that
   (a) $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$ (6%)
   (b) $x_1 < 8$ and $x_2 > 8$. (6%)

9. Find (a) $2^{344} \bmod 11$, (b) $2^{344} \bmod 31$. (10%)

10. Answer these questions for the poset $(\{2, 4, 6, 9, 12, 18, 27, 36, 48, 60, 72\}, |)$. (8%)
   (a) Find the maximal elements.
   (b) Find the minimal elements.
   (c) Is there a greatest element ?
   (d) Is there a least element ?
   (e) Find all upper bounds of {2, 9}.
   (f) Find the least upper bound of {2, 9}, if it exists.
   (g) Find all lower bounds of {60, 72}.
   (h) Find the greatest lower bound of {60, 72}, if it exists.