科目：　微積分　　　　　系所組：　統計資訊系應用統計碩士班

一． 計算下列各題:　　　　　　　　　　　　　　　　　　　　　　(70 %)

(1)　$\displaystyle\lim_{x \to 27} \frac{x - 27}{\sqrt[3]{x} - 3}$

(2)　$\displaystyle\lim_{x \to 0^+} (1 - 12x)^{3/x}$

(3)　$\displaystyle\int \frac{(\sqrt{x} + 3)^5}{\sqrt{x}}\, dx$

(4)　$\displaystyle\int e^{2x} \cos x\, dx$

(5)　$\displaystyle\int_{-\infty}^{\infty} e^{-5|x|}\, dx$

(6)　$\displaystyle\int_0^{16} \int_{\sqrt{y}}^4 \sqrt{x^3 + 4}\, dx dy$

(7)　Find　$f'(e)$　if　$f(x) = \dfrac{(\ln x)^2}{3x}$.

(8)　Find　$\dfrac{dy}{dx}$　at　$(1, 2)$　if　$x^5 y + xy^3 = 10$.

(9)　Find　$f'(2)$　if　$f(x) = \displaystyle\int_{x^2}^{10} \sqrt{2t^3 - 7}\, dt$.

(10)　Find the area of the region completely enclosed by the graphs of the functions $f(x) = x^3 - 3x + 3$ and $g(x) = x + 3$.

二． A truck gets $600/x$ miles per gallon (mpg) when driven at a constant speed of x miles per hour (mph) between 50 and 80 mph. If the price of fuel is \$3/gallon and the driver is paid \$18/hour, at what speed between 50 and 80 mph is it most economical to drive ? (10 %)

三． Use the method of Lagrange multipliers to find the minimum of the function $f(x, y, z) = 2xy + 6yz + 8xz$ subject to the constraint $xyz = 12000$. (10 %)

四． A money market fund has a continuous flow of money at a rate of $f(x) = 1500 - 300x$, reaching 0 in 5 years. Find the present value of this flow if interest is 10% compounded continuously. (10 %)