

科目：統計學

系所組：統計資訊學系應用統計碩士班

1. (15 %) A pair of fair dice are rolled.
- What is the probability that the second die lands on a higher value than does the first?
 - What is the conditional probability that at least one lands on 6 given that the dice land on different numbers?
 - Let X equal the product of the 2 dice. Find the probability distribution for X .

2. (15 %) Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \begin{cases} k(1 - y_2) & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

- Find the value of k that makes this a probability density function.
 - Find $P(Y_2 \geq 3/4 | Y_1 = 1/2)$.
3. (30 %) Let Y_1, Y_2, \dots, Y_n denote a random sample from the normal distribution with unknown mean μ and unknown variance σ^2 .

- Find the method of moments estimators of μ and σ^2 .
- Suppose that μ is known, find the most powerful α -level test of $H_0: \sigma^2 = \sigma_0^2$ versus $H_1: \sigma^2 = \sigma_1^2$, where $\sigma_1^2 > \sigma_0^2$
- Suppose $\mu = 0$ and $\sigma = 1$, and for a fixed x , set

$$X = \begin{cases} Y_1 & \text{if } Y_1 > x \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X)$.

4. (30 %) Let Y_1, Y_2, \dots, Y_n be independent, uniformly distributed random variables on the interval $[0, \theta]$.
- Find a sufficient statistic for θ .
 - Show that $U = (1/\theta)Y_n$ is a pivotal quantity, and find a 95% lower confidence bound for θ .
 - If $\theta = 1$, find the probability density function for $V = Y_1 + Y_2$.
5. (10 %) State and prove the Central Limit Theorem.

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。