1. (14 %) Find the following limits.

    (a) $\lim_{x \to 0} \frac{\sqrt{1-x} - \sqrt{1+x}}{2x}$

    (b) $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{1}{n+i}$

2. (21 %)

    (a) Find $\int \sqrt{1 - x^2}\, dx$

    (b) Evaluate $\int_0^1 \frac{1}{x^{0.2011}}\, dx$

    (c) Evaluate $\int \int_{\Omega} \frac{1}{1+x^2+y^2}\, dx\, dy$, where $\Omega = \{(x,y) \mid x^2 + y^2 \le 1\}$

3. (10 %) State and prove the Mean Value Theorem.

4. (10 %) Use <u>Newton's method</u> to approximate $\sqrt[3]{22}$, continuing until two successive iterations agree to three decimal places.

5. (10 %) Suppose the profit function of a company selling x units of one product and y units of another product is
$$P(x,y) = 12x + 9y - 450 - 0.01(4x^2 + xy + y^2).$$
Find the pair (x, y) that maximizes the company's profit and determine the maximum profit.

6. (10 %) A company has a production function with three inputs x, y and z given by
$$f(x,y,z) = 50x^{2/5}y^{1/5}z^{1/5}.$$
The total budget is \$24000 and the company can by x, y and z at \$80, \$12 and \$10 per unit, respectively. What combination of inputs will maximize production?

7. (15%)

    (a) Find the Taylor series of $f(x) = \sin x$ at $x = 0$.

    (b) Use (a) to find the approximate value of $\int_0^1 \frac{\sin x}{x}\, dx$ up to the third decimal place.

8. (10%) Solve the differential equation and initial condition
$$\begin{cases} y' = 6x - 2xy \\ y(0) = 2 \end{cases}$$