(100)輔仁大學碩士班招生考試試題	考試日期：100 年 3 月 18 日第 貳 節
	本試題共 肆 頁 (本頁為第 壹 頁)

科目： 數學　　系所組： 資工系

1. (8%) Show that the set $\{0^n 1^n | n \in \mathbf{Z}^+ \cup \{0\}\}$ is not regular.

2. (12%) For the alphabet $\Sigma = \{0,1\}$, express the following set using a regular expression

(a) the set of strings containing exactly 1.

(b) the set of strings of odd length.

(c) the set of strings ending in 1 and not containing 000.

3.

(a) (4%) Solve the congruence $4x \equiv 5 \pmod 9$.

(b) (6%) Show that an inverse of a modulo m does not exist if $\gcd(m, a) > 1$.

4. (12%) Solve the following recurrence relation

(a) $\begin{cases} a_n = 3a_{n-1} + 2b_{n-1} \\ b_n = a_{n-1} + 2b_{n-1} \end{cases}$, and $\begin{cases} a_0 = 1 \\ b_0 = 2 \end{cases}$

(b) $a_n = 5a_{n-1} + 6a_{n-2} + 7^n$, with $a_0 = 1, a_1 = 2$.

5. Find the coefficient of x^9 in the following power series.

(a) (3%) $1/(1+3x)$

(b) (5%) $x^4/(1-x)^3$

※注意：還有第貳頁.

※ 注意：1.考生須在「彌封答案卷」上作答。
2.本試題紙空白部份可當稿紙使用。
3.考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。

科目： 數學 系所組： 資訊工程學系

(10%) 6. Consider a linear system $\mathbf{Ax} = \mathbf{b}$, where $\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 5 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, and $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

What is the reduced echelon form of the augmented matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 3 & 3 & 0 \\ 1 & 3 & 5 & 2 \end{bmatrix}$ after forward elimination, and what is the solution of the linear system?

(10%) 7. Find all the eigenvalues and eigenvectors of $\mathbf{A} = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$.

(12%) 8. Suppose that **A** is an m by n matrix of rank r. Write all known relations (=, < or ≤) between r, m and n.

(a) If $\mathbf{Ax} = \mathbf{b}$ has infinitely many solutions for every **b**. (4%)

(b) If $\mathbf{Ax} = \mathbf{b}$ has exactly one solution for some **b**, no solution for other **b**. (4%)

(c) If $\mathbf{Ax} = \mathbf{b}$ has exactly one solution for every **b**. (4%)

(18%) 9. Answer each of the following questions.

(a) Let **P** be the plane in $\mathbf{R}^3$ space with equation $3x + 2y - 6z = 8$. What is the equation of the plane $\mathbf{P}_0$ through the origin parallel to **P**? Are **P** and $\mathbf{P}_0$ subspaces of $\mathbf{R}^3$? (6%)

(b) Let matrix $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$. Describe the column space and the null space of **A**. (6%)

(c) If a 2 by 2 matrix **A** has an eigenvalue $\lambda_1 = 1$ with eigenvector $\mathbf{x}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and the other eigenvalue $\lambda_2 = 5$ with eigenvector $\mathbf{x}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, what is the matrix **A**? (6%)

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