科目：工程數學 (C) (線性代數、離散數學)　　　系所組：電機工程學系（丙組）

(線性代數) (50%)

1. Use Gramer's rule to solve: (10%)

$$\begin{cases} x_1 + 2x_2 + x_3 = 5 \\ 2x_1 + 2x_2 + x_3 = 6 \\ x_1 + 2x_2 + 3x_3 = 9 \end{cases}$$

2. In changing coordinates, let $u_1 = (3, 2)^T$, $u_2 = (1, 1)^T$, and x = $(7, 4)^T$. Find the coordinates of x with respect to u_1 and u_2. (10%)

3. In row space and column space, find the dimension of the subspace of $\mathbf{R^4}$ spanned by: (5%)

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 0 \end{bmatrix}, \quad x_2 = \begin{bmatrix} 2 \\ 5 \\ -3 \\ 2 \end{bmatrix}, \quad x_3 = \begin{bmatrix} 2 \\ 4 \\ -2 \\ 0 \end{bmatrix}, \quad x_4 = \begin{bmatrix} 3 \\ 8 \\ -5 \\ 4 \end{bmatrix}$$

4. In linear transformations, let L be the linear transformation mapping $\mathbf{R^3}$ into $\mathbf{R^2}$ defined by:

$$L(x) = x_1 b_1 + (x_2 + x_3) b_2$$

for each $x \in \mathbf{R^3}$, where

$$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Find the matrix A representing L with respect to the ordered bases $\{e_1, e_2, e_3\}$ and $\{b_1, b_2\}$. (5%)

5. Find the equation of the plane that passes through the points: $P_1 = (1, 1, 2)$, $P_2 = (2, 3, 3)$, and $P_3 = (3, -3, 3)$. (5%)

6. In norms of inner product spaces, let x be the vector $(4, -5, 3)^T$ in $\mathbf{R^3}$. Compute $\|x\|_1$, $\|x\|_2$, and $\|x\|_\infty$. (6%)

7. In exponential of a matrix, compute e^A for: (9%)

$$A = \begin{bmatrix} -2 & -6 \\ 1 & 3 \end{bmatrix}$$

(離散數學) (50%)

1. Travis tosses a fair coin twice. Then he tosses a biased coin, one where the probability of a head is 3/4, four times. What is the probability Travis's six tosses result in five heads and one tail? (5%)

2. In plain and one-to-one functions, if there are 2187 functions $f: A \to B$ and $|B| = 3$, what is $|A|$? (4%)

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3. In special functions, let $A = \{x, a, b, c, d\}$.
   (a) How many closed binary operations f on A satisfy $f(a, b) = c$? (5%)
   (b) How many of the functions f in part (a) have an identity? (5%)
   (c) How many of the functions f in part (b) are commutative? (5%)

4. In computational complexity, use the results of the following Table to determine the best "big-Oh" form for each of the following functions $f: \mathbf{Z}^+ \to \mathbf{R}$.

   (a) $f(n) = 3 + \sin(1/n)$. (5%)

   (b) $f(n) = 5n^2 + 3n \log_2 n$. (5%)

Big-Oh Form	Name
$O(1)$	Constant
$O(\log_2 n)$	Logarithmic
$O(n)$	Linear
$O(n \log_2 n)$	$n \log_2 n$
$O(n^2)$	Quadratic
$O(n^3)$	Cubic
$O(n^m), m = 0, 1, 2, 3, \ldots$	Polynomial
$O(c^n), c > 1$	Exponential
$O(n!)$	Factorial

5. If $A = \{1, 2, 3, 4, 5\}$ and R is the equivalence relation on A that induces the partition $A = \{1, 2\} \cup \{3, 4\} \cup \{5\}$, what is R? (5%)

6. (a) How many spanning subgraphs are there for the graph (G) in the following Figure? (2%)
   (b) How many connected spanning subgraphs are there in part (a)? (2%)
   (c) How many of the spanning subgraphs in part (a) have vertex a as an isolated vertex? (2%)

   (G)
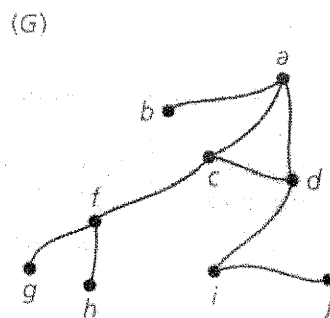
7. About weighted trees and prefix codes, a code for {a, b, c, d, e} is given by a: 00　b: 01　c: 101 d: $x10$　e: $yz1$, where $x, y, z \in \{0, 1\}$. Please determine x, y, and z so that the given code is a prefix code. (5%)