科目：工程數學(A)(線性代數.機率學)系所組：電機工程學系甲組

Part 1: 線性代數

1. Consider the matrix

$$A = [a_1 \ a_2 \ a_3]$$
$$a_1 = [2 \ 1 \ 1]^T, \ a_2 = [-2 \ 1 \ 3]^T, \ a_3 = [3 \ 1 \ -1]^T$$

   (a) If A^{-1} exists, find the eigenvalues of A^{-1}.
   (b) Show that the eigenvectors of A are linearly independent.
   (c) What is the corresponding diagonal matrix of A?
   (d) Find the eigenvalues and eigenvectors of $A^4 - 3A^2 - 2I$.

   (30%)

2. Find the basis of the matrix

$$T = \begin{bmatrix} a+b+2c+3d & a+b+2c+3d \\ 2a+2c+4d & a+b+2c+3d \end{bmatrix} \quad a, b, c, d \in \Re$$

   (15%)

3. Given a matrix as

$$A = \begin{bmatrix} 2 & 3 & 6 \\ 6 & 2 & -3 \\ 3 & -6 & 2 \end{bmatrix}$$

   Find a constant c such that cA is orthogonal.

   (15%)

Part 2: 機率

1. Find the probability of the following problems:
   (a) Toss five dices, and find the probability of each dice showing different values.
   (b) Toss two dices, and find the probability of the total value is bigger than 5.

   (20%)

2. Suppose X is uniformly distributed on the range of [-2, 2]

(a) Find the probability of $X^2 = 1$.

(b) Find the probability of $X^2 \geq 1$.

(10%)

3. Choose two random numbers: $-1 \leq X \leq 1$ and $-2 \leq Y \leq 2$. Find the probability of $-1 \leq XY \leq 1$.

(10%)

※ 注意： 1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部分可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。