

科目： Advanced Calculus 高等微積分系所組：

數學所

1 State the following theorems:

(1). Fundamental Theorem of Calculus. (10分)

(2). Intermediate Value Theorem. (10分)

2 State the following definitions:

(1). A function  $f : D(f) \subseteq \mathbf{R} \rightarrow \mathbf{R}$  with  $[a, b] \subseteq D(f)$ , is integrable on  $[a, b]$ . (10分)(2). A function  $f : D(f) \subseteq \mathbf{R} \rightarrow \mathbf{R}$  with  $c \in D(f)$ , is continuous at  $c$ . (10分)3 Show that  $\sum_{k=0}^{\infty} (-1)^k \frac{1}{k+1}$  converges. Find its sum. (10分)4 Let  $f : I \rightarrow \mathbf{R}$  be differentiable on an interval  $I$  in  $\mathbf{R}$ . Prove that  $f$  is uniformly continuous on  $I$ , if its derivative  $f'$  is bounded on  $I$ . (10分)5 Let  $f : \mathbf{R}^2 \rightarrow \mathbf{R}$  be defined by  $f(x, y) = \begin{cases} \frac{x^2 - xy}{x+y} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$  Find the partial derivatives  $f_x(0, 0)$  and  $f_y(0, 0)$  of  $f$  at  $(0, 0)$ . (10分)6 Find the limit  $\lim_{k \rightarrow \infty} \frac{\ln(1 + \frac{1}{k})}{\sin \frac{1}{k}}$ . (10分)7 Determine whether  $\sum_{k=1}^{\infty} \frac{k^2 \sin kx}{k^4 + x^2}$  converges uniformly on  $\mathbf{R}$ . (10分)8 Determine whether the improper integral  $\int_0^{\infty} \frac{\sin x}{\sqrt{x^2 + x + 1}} dx$  converges or diverges. (10分)

※ 注意：1. 考生須在「彌封答案卷」上作答。

2. 本試題紙空白部份可當稿紙使用。

3. 考生於作答時可否使用計算機、法典、字典或其他資料或工具，以簡章之規定為準。