考試科目	線性代數 81112、81162	系所別	應用數學系	考試時間	2月19日(日)第二節

Please show all your work.

1. **(15%)** Let $A=[a_{i,j}]$ be a symmetric tridiagonal matrix (i.e. A is symmetric and $a_{i,j}=0$ whenever $1<|i-j|$). Let $M_{i,j}$ denote the matrix obtained from A by deleting the row and column containing $a_{i,j}$ and let B be the matrix obtained from A by deleting the first two rows and columns. Show that

$$\det(A)=a_{1,1}\det(M_{1,1})-a_{1,2}^2\det(B).$$

2. **(15%)** Let $P_2(R)$ denote the space of all polynomials with coefficients in R having degree less than or equal to 2. Let $T:P_2(R)\to P_2(R)$ be defined by $T(f(x))=f(x)+f'(x)+f''(x)$, where $f'(x)$ and $f''(x)$ denote the first and second derivatives of $f(x)$. Either show that T is not invertible or find its inverse.

3. **(15%)** A matrix A is said to be idempotent if $A=A^2$. Show that $I+A$ is nonsingular if A is idempotent.

4. **(15%)** Let $S=\{(1,0,1),(0,1,1),(1,3,3)\}$ be a subset of the inner product space R^3 over the field R. Apple the Gram-Schmidt process to obtain an orthonormal basis β for span(S).

5. **(20%)** Let x,y be nonzero column vectors in R^n with $n\ge 2$ and let $A=xy^T$. Show that (a) $\lambda_1=0$ is an eigenvalue of A with $n-1$ linearly independent eigenvectors and (b) A is diagonalizable if $x^Ty\neq 0$.

6. **(20%)** Let T be a linear operator on a vector space V, and suppose that V is a $T-$cyclic subspace of itself. Prove that if U is a linear operator on V, then $UT=TU$ if and only if $U=g(T)$ for some polynomial $g(t)$.

備註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。