

考試科目	微積分 81111, 81161	系所別	應用數學系	考試時間	2月19日(星期日)第一節
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1. Evaluate the limits.

$$(a) (6\%) \lim_{\theta \rightarrow 0} \frac{\tan(2\theta)(1 - \cos\theta^2)}{\theta^2 \sin^3(3\theta)}. \quad (b) (6\%) \lim_{x \rightarrow \infty} \left(x - x^2 \ln\left(\frac{1+x}{x}\right) \right).$$

2. Evaluate the integrals.

$$(a) (8\%) \int \cos(\ln x) dx.$$

$$(b) (8\%) \int \frac{1}{x^2 - 2x} dx.$$

$$(c) (8\%) \int_2^{\infty} \frac{\sqrt{x^2 - 4}}{x^3} dx.$$

$$(d) (8\%) \int_0^{\frac{\pi}{2}} \frac{1}{1 + \sin^2 x} dx.$$

3. Determine if each series converges or diverges.

$$(a) (8\%) \sum_{n=1}^{\infty} \frac{(n+1)3^n}{2^{2n+1}}. \quad (b) (8\%) \sum_{n=1}^{\infty} \ln\left(\frac{n}{3n+1}\right).$$

4. (8%) Find the values of α such that the following integral is convergent.

$$\int_0^{\infty} \frac{\sin x}{x^\alpha} dx.$$

5. (8%) Find the arc length of the curve $r = 1 + \sin \theta$ where $\theta \in [0, 2\pi]$.

6. (8%) Let $p_k \in [0, 1]$, $k = 1, 2, \dots, 10$ with $\sum_{k=1}^{10} p_k = 1$. Find the maximum of $\sum_{k=1}^{10} (2p_k - 3p_k^2)$.

7. (8%) Find the double integral $\iint_R \frac{x-2y}{3x-y} dA$ where R is the parallelogram enclosed by the lines $x - 2y = 0$, $x - 2y = 4$, $3x - y = 1$ and $3x - y = 8$.

8. (8%) Find the line integral $\int_C \tan y dx + x \sec^2 y dy$ where $C : (t^2, \frac{\pi t}{4})$ from $t = 0$ to $t = 1$.

備註

一、作答於試題上者，不予計分。
二、試題請隨卷繳交。