

系級	數學系碩士班	考試時間	100 分鐘
科目	線性代數	本科總分	100 分

- (20 points) Let V be the set of all real polynomials of degree at most 2. Prove that $\{1, x+1, (x+1)^2\}$ is a basis for V .
- (20 points) Let A be an $n \times n$ real matrix. Suppose λ is an eigenvalue of A then the complex conjugate $\bar{\lambda}$ is an eigenvalue of A . Give an example to show that the result is false if A is a complex matrix.
- (20 points) Let A, B, C be $m \times n$, $n \times p$, $p \times q$ matrices respectively. Prove $A(BC) = (AB)C$.
- (20 points) Consider the matrix

$$A = \begin{pmatrix} 0 & 6 & 6 & 3 \\ 1 & 2 & 1 & 1 \\ 4 & 1 & -3 & 4 \\ 1 & 3 & 2 & 0 \end{pmatrix}.$$

Find a basis for the column space of A , and the rank of A .

- (20 points) Diagonalize the matrix

$$A = \begin{pmatrix} -3 & 5 \\ -2 & 4 \end{pmatrix},$$

and compute A^k in terms of k .