東吳大學100學年度碩士班研究生招生考試試題

系級	數學系碩士班	考試時間	100 分鐘
科目	高等微積分	本科總分	100 分

1. (10%) Evaluate the integral $\int_0^{\pi}\int_0^{\pi} y\cos(xy)\,dy\,dx$.

2. (10%) Prove that $f(x)=\begin{cases}\dfrac{x}{1+e^{1/x}} & x\neq 0\\ 0 & x=0\end{cases}$ is differentiable on $[0,\infty)$.

3. (10%) Prove that the function $f(x)=\begin{cases}\cos\dfrac{1}{x} & x\neq 0\\ 0 & x=0\end{cases}$ has no limit as $x\to 0$.

4. (10%) Prove that $1+x+\dfrac{x^2}{2!}+\cdots+\dfrac{x^n}{n!}<e^x$ for every $x>0$ and every $n\in N$.

5. (10%) Let $a_0, a_1, \ldots$ be a sequence of real numbers. If $a_k\to a$ as $k\to\infty$, does $\sum_{k=1}^{\infty}(a_{k+1}-2a_k+a_{k-1})$ converges? If so, what is the value?.

6. . (15%) Let $f: R\to R$ is continuous and $F(x)=\int_0^x f(t-x)\,dt$.

(a) (5%) State the fundamental theorem of Calculus.
(b) (10%) Find $F'(x)$.

7. (15%) Let $f: R^2\to R$ be a function defined by $f(x,y)=\sqrt{|xy|}$.

(a) (5%) State the definition of that f is differentiable at (x_0,y_0).
(b) (10%) Find $\dfrac{\partial f}{\partial x}(0,0), \dfrac{\partial f}{\partial y}(0,0)$ and prove that f is not differentiable at $(0,0)$.

8. (20%) Prove the following:

(a) (10%) If I is a closed, bounded interval and $f: I\to R$ is a continuous function, then f is uniformly continuous on I.
(b) (10%) $f(x)=x\ln x$ is uniformly continuous on $(0,1)$.