國立臺灣海洋大學 106學年度研究所碩士班招生考試試題

考試科目：自動控制及線性代數

系所名稱：通訊與導航工程學系碩士班控制組、通訊與導航工程學系碩士班電子導航與定位組、電機工程學系碩士班控制組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

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1. **(15%)** Consider a unity feedback system with the forward transfer function

$$G(s) = \frac{10K}{s(0.1s+1)}$$

- Compute the error constants K_p, K_v, and K_a.
- Compute the steady-state error $e_{ss}(step)$ and $e_{ss}(ramp)$.

2. **(15%)** Consider a unity feedback system with the forward transfer function

$$G(s) = \frac{Ks}{s^2 - 2s + 1}$$

- Sketch the root locus and find the $j\omega$-axis crossing.
- Compute the break-in point.

3. **(20%)** Consider a unity feedback system with the forward transfer function

$$G(s) = \frac{K}{s(s+2)(s+4)}$$

- Use Nyquist plot and stability criterion to find the stabilizing range of K.
- Verify your result with Routh stability criterion.

4. $B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & -0.5 \end{bmatrix}$. (1) Find the eigenvalues and corresponding eigenvectors for matrix B. (2) Find $rank(B)$. (3) Find $\det(B)$. (10%)

5. Suppose that λ is an eigenvalue of a square matrix B. (1) Prove that e^{λ} is an eigenvalue of matrix e^{B}. (2) Show that e^{B} is nonsingular for any square matrix B. (10%)

6. (1) Show that the vectors $v_1 = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ are linearly independent. (2) Suppose that $v = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $v = a_1 v_1 + a_2 v_2 + a_3 v_3$. Find $a_1, a_2,$ and a_3. (10%)

7. $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 3 \\ 1 & 0 & 2 \end{bmatrix}$. (1) Find a basis for the null space of matrix B. (2) Find a basis for the range space of matrix B. (10%)

8. Suppose that B is a real matrix and $B = -B^{T}$. Prove that the real part of any eigenvalue λ of matrix B is 0 (i.e., $Re(\lambda) = 0$). (10%)