

國立臺灣海洋大學 106學年度研究所碩士班招生考試試題

考試科目：計算機數學（含線性代數、離散數學）

系所名稱：資訊工程學系碩士班不分組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

1. (6%) Given a matrix A as $A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -3 & 5 \\ 1 & 0 & 12 \end{bmatrix}$, find a matrix B satisfying $AB = -A^2 + 3A$.

2. (13%) Given the following matrix A , Find: (3%)(a) rank(A)

(5%)(b) a basis for the row space of A .

(5%)(c) a basis for the nullspace of A .

$$A = \begin{bmatrix} 2 & 4 & 1 & -1 & 2 \\ -1 & -2 & 3 & 0 & -2 \\ 0 & 0 & 1 & 8 & -4 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

3. (7%) Find the determinant of the given matrix.

$$\begin{bmatrix} 5 & 2 & 4 & 0 \\ 2 & -3 & -1 & 2 \\ 3 & -4 & 3 & 7 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

4. (14%) Given $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$, 4%(a) Is A diagonalizable? why?

10%(b) find the eigenvectors of A .

5. (10%) Determine each of the following statements True or False.

a. () If A is invertible, then $A + A^T$ is invertible.

b. () Every matrix is row equivalent to a unique matrix in reduced row-echelon form.

c. () A linear system with fewer equations than unknowns may have no solutions.

d. () The row space of AC is contained in the row space of C . (the product AC is defined)

e. () There is a unique coordinate vector associated with each vector $v \in V$,

where V is non-zero finite-dimensional vector space.

6. $|A| = 7, |B| = 4$. (15%)

- (a) How many functions are there from A to B?
- (b) How many 1-1 functions are there from A to B?
- (c) How many onto functions are there from A to B?

7. How many non-negative integer solutions $x_1 + x_2 + x_3 \leq 100$ (10%)

8. Solve the recurrence relation $a_n = 6a_{n-1} - 8a_{n-2} + 3^n, a_1 = 1, a_2 = 2$ (10%)

9. True or False. If R is transitive $\Rightarrow R^2$ is transitive? why?
(15%)