國立臺灣海洋大學 106學年度研究所碩士班招生考試試題

考試科目：工程數學

系所名稱：輪機工程學系碩士班不分組

1. 答案以橫式由左至右書寫。2. 請依題號順序作答。

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1. (15%) Determine whether the given differential equation is exact. If it is exact, solve it.

$$(\sin y - y\sin x)dx + (\cos x + x\cos y - y)dy = 0$$

2. (15%) Solve the given differential equation by variation of parameters.

$$x^2y'' + xy' - y = \ln x$$

3. (10%) Use the Laplace transform to solve the given boundary-value problem.

$$y'' + 2y' + y = 0, \qquad y'(0) = 2, \qquad y(1) = 2$$

4. (10%) Use the Laplace transform to solve the given integrodifferential equation.

$$y'(t) = 1 - \sin t - \int_0^t y(\tau)d\tau, \qquad y(0) = 0$$

5. (20%) Find the (a) determinant (b) inverse matrix (c) eigenvalues and (d) eigenvectors of the given nonsingular matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 4 & 0 & 1 \end{bmatrix}$$

6. (10%) Use the Gram-Schmidt orthogonalization process to transform the given basis $B = \{< 1,1,0 >, < 1,2,2 >, < 2,2,1 >\}$ for $\boldsymbol{R}^3$ into an orthogonal basis B' and an orthonormal basis B''.

7. (10%) Use Green's theorem to evaluate the given line integral

$$\oint_C xydx + x^2dy,$$

where C is the boundary of the region determined by the graphs of $x = 0,\ x^2 + y^2 = 1, x \geq 0.$

8. (10%) Find the half-range cosine and sine expansions of the given function.

$$f(x) = \begin{cases} 1, & 0 < x < 1/2 \\ 0, & 1/2 \leq x < 1 \end{cases}$$