1. (5% each, totally 10%) A box contains a fair coin and a two-headed coin.
   (a) A student randomly draws one of the coins from the box, what is the probability that it is the fair coin?
   (b) Suppose that the student flips the same coin and it shows head. What is the probability that it is the fair coin?

2. (5% each, totally 25%) Today a machine will produce 100 resistors in order. Suppose that a bad resistor is produced independently with the probability of $p$. Let $N$ be the number of resistors before the first bad resistor is produced (i.e., $N$ does NOT include the first bad resistor). Let $N_b$ be the total number of bad resistors.
   (a) Is $N$ a discrete random variable or continuous random variable?
   (b) What is the probability mass function (PMF) of $N$?
   (c) What is the cumulative distribution function (CDF) of $N$?
   (d) What is the PMF of $N_b$?
   (e) What is the expectation of $N_b$?

3. (5% each, totally 15%) Let $X$ and $Y$ be random variables, with the joint PMF

$$P_{X,Y}(x,y) = \begin{cases} 1/4 & \text{if } (x,y) = (1,1),(1,2),(2,1),(2,2); \\ 0 & \text{else.} \end{cases}$$

   (a) Let $W = \max(X,Y)$. What is the PMF of $W$?
   (b) What is the CDF of $W$?
   (c) What is the expectation of $W$?

4. (5%) $F_X(x)$ is a cumulative distribution function (cdf). Is it true that $F_X(\infty) = 1$ ? (Answer yes or no).

5. (5% each, totally 10%) The joint cdf for the vector of random variable X = (X,Y) is given by

$$F_{X,Y}(x,y) = \begin{cases} (1 - e^{-\alpha x})(1 - e^{-\beta y}) & x \geq 0, y \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) Find the marginal cdf's.
   (b) Find the probability of the event A= { $X > x$, $Y > y$}.

6. (5% each, totally 10%) Let $Y = a\cos(\omega t + \Theta)$ where $a$, $\omega$ and $t$ are constants, and $\Theta$ is a uniform random variable in the interval $(0, \pi)$. The random variable Y results from sampling the amplitude of a sinusoid with random phase $\Theta$.
   (a) Find the expected value of $Y$.
   (b) Find the variance of $Y$.

7. (5% each, totally 15%) A random variable X has a probability density function (pdf):

$$f_X(x) = \begin{cases} c(1 - x^4) & -1 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

   (a) Find c.
   (b) Find cdf of X.
   (c) Find the variance of X.

8. (5% each, totally 10%) Let $\alpha$ be a point selected at random from the interval (0,1). Consider the random variable $X = \sqrt{1 - \alpha}$.
   (a) Find cdf of X.
   (b) Plot pdf of X.