

# 國立臺北大學 106 學年度碩士班一般入學考試試題

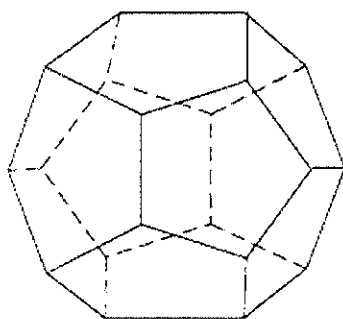
系（所）組別：資訊工程學系

科目：線性代數與離散數學

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☐可 ☒不可使用計算機

1. (10%) Do the following computations.
  - (a) Convert the binary number  $(11010.101)_2$  into decimal. (5%)
  - (b) Convert the decimal number 25.6 into binary. (5%)
2. (16%) Boolean function  $f(x, y, z, w) = \neg x \vee (x \wedge y \wedge z \wedge w) \vee (x \wedge y \wedge z \wedge \neg w)$ . First, find the truth table of function  $f$ . Next, simplify the expression of  $f$  to minimize the number of AND, OR, NOT operations. Then, draw out the combinatorial circuit corresponding to the simplified expression. Finally, transform and **redraw** the above combinatorial circuit to contain NAND gates only.
3. (12%) Consider the following graph. It has 20 vertices each with degree 3.
  - (a) Is it a planar graph? Justify your answers. (4%)
  - (b) Does it have an Euler path? Justify your answers. (4%)
  - (c) Does it have a Hamilton path? Justify your answers. (4%)



4. (12%) Consider the following relations on  $\{1, 2, 3\}$ .  $R_1 = \{(1,1), (1,3), (2,2), (3,1)\}$ ,  $R_2 = \{(1,1), (2,2), (3,1), (3,3)\}$ ,  $R_3 = \{(1,2), (2,1), (3,3)\}$ ,  $R_4 = \{(1,3), (2,3)\}$ .
  - (a) Which of these relations are reflexive? Justify your answers. (4%)
  - (b) Which of these relations are **antisymmetric**? Justify your answers. (4%)
  - (c) Which of these relations are transitive? Justify your answers. (4%)
5. (50%) Give the matrix  $\mathbf{M} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ . Solve the following questions.
  - (a) (10%) Describe the nullspace of  $\mathbf{M}$ . (The nullspace of  $\mathbf{M}$  consists of all solutions to  $\mathbf{M}\mathbf{x} = \mathbf{0}$ .)
  - (b) (5%) Compute  $\mathbf{M}^T\mathbf{M}$  and  $\mathbf{M}\mathbf{M}^T$ .
  - (c) (5%) Explain whether  $\mathbf{M}^T\mathbf{M}$  is similar to  $\mathbf{M}\mathbf{M}^T$ .
  - (d) (10%) Compute the eigenvalues and eigenvectors of  $\mathbf{M}^T\mathbf{M}$ .
  - (e) (10%) Diagonalize  $\mathbf{M}^T\mathbf{M}$ .
  - (f) (10%) Find the singular value decomposition of  $\mathbf{M}$ .

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