

# 國立臺北大學 106 學年度碩士班一般入學考試試題

系（所）組別：統計學系  
科 目：數理統計

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☐可 ☒不可使用計算機

1. (50pt) Let  $X_1, X_2, \dots, X_n$  be iid  $\text{Uniform}(a, b)$  where  $a < b$ .

a.(5pt) Find the moment-generating-function of  $X_1$ .

b.(5pt) Let  $\bar{X}_n = \sum X_i / n$ . Find the variance of  $\bar{X}_n$ .

c.(10pt) Find the distribution of  $X_1 + X_2$ .

d.(10pt) Let  $Y_1 = \min\{X_1, X_2, \dots, X_n\}$ . Find the pdf of  $Y_1$  and its variance.

e.(7pt) Let  $Z = -\log(\frac{X_1 - a}{b - a})$ . Find the distribution of  $Z$ .

f.(6pt) Let  $W = \begin{cases} a, & \text{if } a < X_1 < \frac{a+b}{2} \\ b, & \text{if } \frac{a+b}{2} \leq X_1 < b \end{cases}$ . Find the mean and variance of  $W$ .

g.(7pt) Find the mean and variance of  $X_1$  given  $W$ .

2. (50pt) Let  $X_1, X_2, \dots, X_n$  be a random sample from the distribution with a probability density function

$$f(x) = \frac{2x}{\theta^2}, \quad 0 < x < \theta,$$

where  $\theta > 0$ .

a. Find the maximum likelihood estimator  $T_n(\mathbf{X})$  of  $\theta$ , where  $\mathbf{X} = (X_1, X_2, \dots, X_n)'$ .

b. Based on (a), obtain an unbiased estimator  $U_n(\mathbf{X})$  of  $\theta$ .

c. Show that  $U_n(\mathbf{X})$  is the unique minimum variance unbiased estimator of  $\theta$ .

d. Determine the likelihood ratio test  $\Lambda$  for testing  $H_0: \theta = \theta_0$  versus  $H_a: \theta \neq \theta_0$ , where  $\theta_0 > 0$  is a constant. When  $H_0$  is true, find the distribution of  $-2 \log \Lambda$ .

e. Find a test and the corresponding critical value for testing  $H_0: \theta \leq 0.5$  versus  $H_a: \theta > 0.5$  so that the significance level is  $\alpha = 0.05$  and compute the power at  $\theta = 0.6$ .