國立臺北大學 106 學年度碩士班一般入學考試試題

系（所）組別：經濟學系
科　　　目：統計學

☑可 ☐不可使用計算機

I. (50 points) Fill in the following blanks.

1. We have a one-independent variable regression model: $y_i = \alpha + \beta x_i + \varepsilon_i$, with ε_i following iid $N(0, \sigma^2)$. $\hat{\alpha}$ and $\hat{\beta}$ are least squares estimates of α and β, respectively. Suppose $z_1 = \sqrt{\sum(x_i - \bar{x})^2}(\hat{\beta} - \beta)$, then $Var(z_1) =$ ___(1)___.

2. We run an OLS regression of the form $\hat{y} = \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2$, where y = executive salaries, x_1 = sales and x_2 = profits, across a sample of 102 firms. The result is $\hat{y} = 0.5x_1 + 0.4x_2$, and we know $\sum_i x_{1i}x_{2i} = 8$, $\sum_i x_{1i}^2 = \sum_i x_{2i}^2 = 10$, $Var(\hat{\beta}_1) = Var(\hat{\beta}_2) = 0.7$ and $Cov(\hat{\beta}_1, \hat{\beta}_2) = -0.5$. All variables are expressed as <u>deviations</u> about their means. Suppose we first regress profits on sales, and obtain the residuals x_2^*, i.e., $x_2 = \hat{\alpha}x_1 + x_2^*$. Second, we regress y on x_1 and x_2^* to have $\hat{y} = \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2^*$. Then, we have $\hat{\alpha} =$ ___(2)___ and $\tilde{\beta}_1 =$ ___(3)___.

3. Suppose *price* is house price, *assess* is the assessed housing value (before the house was sold), *lotsize* is size of the lot in feet, *sqrft* is square footage, and *bdrms* is number of rooms. In the simple regression model $price = \beta_0 + \beta_1 \, assess + u$, the assessment is rational if $\beta_1 = 1$ and $\beta_0 = 0$. The estimated equation with standard errors listed in the parentheses is

$$\hat{price} = -14.47 + 0.976\,assess, \quad n = 88, \quad SSE = 165{,}644.51, \quad R^2 = 0.820,$$

where *SSE* is the sum of the squared errors. To test the joint hypothesis $H_0: \beta_0 = 0 \text{ and } \beta_1 = 1$, we need the restricted model, which is ___(4)___. Suppose the restricted model's *SSE* equals 209,448.99. Then the *F* statistic for the joint hypothesis is ___(5)___.

4. An investigator estimates the simple normal linear model $y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$ $(i = 1, \ldots, 12)$. By least squares, and reports the conventional 95% confidence interval (0.1772, 0.6228) for $\beta_1 + \beta_2$, and (0.0860, 2.3140) for $\beta_1 - \beta_2$. The critical values of *t* for 10 degrees of freedom are 2.228 and –2.228. Then, the standard errors of $b_1 + b_2$ is ___(6)___.

5. Suppose that you have two independent unbiased estimators of parameter θ, say $\hat{\theta}_1$ and $\hat{\theta}_2$, with different variances v_1 and v_2. The linear combination $\hat{\theta} = c_1\hat{\theta}_1 + c_2\hat{\theta}_2$ is the minimum variance unbiased estimator of θ? Then, $c_1 =$ ___(7)___ (in terms of v_1 and v_2).

6. Consider the following linear model that explains monthly beer consumption of individual *i*:

$$beer_i = \alpha + \beta_1 price_i + \beta_2 educ_i + \beta_3 inc_i + \delta_1 female_i + \varepsilon_i, \quad i = 1, 2, \ldots, n.$$

Suppose that all Gauss-Markov assumptions are satisfied except the conditional variance of error term ε_i equal to $\sigma^2 inc_i^2$. We transform the regression to have a homoscedastic error term. Then, the intercept in the transformed model is ___(8)___.

7. Suppose we have the following model: $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$, $u_t = \rho u_{t-1} + e_t$, where $0 < |\rho| < 1$ and $\{e_t\}$ is an i.i.d. sequence with mean zero and variance σ_e^2. Then, will the OLS estimators of β_0 and β_1 be consistent? ____(9)____ (Yes or No).

8. Using a set of panel data, we estimate the regression of $\bar{y}_i = \beta_0 + \beta_1 \bar{x}_i + u_i + \bar{\varepsilon}_i$, where the overbar represents the average over time, and u_i is the fixed unobserved individual effect. Suppose $E(u_i) = 0$, $Cov(\bar{x}_i, \bar{\varepsilon}_i) = 0$ and $Cov(x_{it}, u_i) = \sigma_{xu}$ for $i = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$. Letting $\tilde{\beta}_1$ be the OLS estimator of β_1 using time-average data, then $plim\,\tilde{\beta}_1 =$ ____(10)____.

II. 填充題：(總分50分，每題5分)

- 答案請標示對應題號 (1) 至 (10)
- 答案可得明確數值者，請寫下該數值並四捨五入進位到小數第四位。

【壹】紐約時報與CBS的民調顯示，川普支持者裡有2/3「支持」暫停7移斯林國國民入境美國的禁令，而非川普支持者裡有百分之87「不支持」該禁令的。此外，據調查美國民眾有百分之46為川普支持者。

a. 隨機抽出一名民眾，得知他支持川普的禁令。請問他是川普支持者的可能性為 (1) 。

隨機抽出N名川普支持民眾，令X_i代表第i位抽出民眾對禁令的態度：其值為1表示支持，為-1表示不支持。考慮隨機變數$\bar{X} = \sum_{i=1}^{N} X_i / N$

b. 寫下X_i的期望值及變異數。 (2)

c. 寫下當$N = 3$時$\bar{X}$的抽樣分配。 (3)

d. 設N=100，寫下$\bar{X}$的大樣本漸近分配（需寫下分配名稱及對應分配參數值） (4)

【貳】隨機變數X出象非0即1，令$\Pr(X = 1) = p$。考慮樣本平均$\bar{X} = \sum_{i=1}^{N} X_i / N$，其中$X_i$代表第i個隨機自X母體抽出的觀察值。

a. 在N=100下，若得到一組樣本且其平均$\bar{x} = 0.2$，寫下對參數p的95%信賴區間估計值： (5)

b. 承a小題，請問若$p = 0.4$，則$E(X_i)$落在該信賴區間的機率是多少？ (6)

c. 在N=100下，考慮如下的區間估計式 $[\bar{X} - 0.02\ ,\bar{X} + 0.04]$，請問若$p = 0.4$，則$E(X_i)$落在該估計區間的機率是多少？ (7)

d. 考慮虛無假設$H_0: p = 0.3$與對立假設$H_1: p = 0.5$，並以$0.48 < \bar{X}$為拒絕虛無假設的條件。當N=100，請問此時的檢定力是多少？ (8)

【參】飛鳶咖啡店設計了線上滿意度問卷，顧客可勾選的滿意度分數為1分，2分或3分。假設顧客消費後的效用水準U服從指數分配，其機率密度函數為$f(u) = \lambda e^{-\lambda u}, u > 0$。若當消費者效用低於4時，他會勾選1分；當介於4與6時，會勾選2分；而大於6時，會勾選3分。

a. 預期平均滿意度分數為何？ (9)

b. 若消費者只有滿意分數為2分或3分時才會上線填答，請問當線上填答樣本很大時，其樣本平均滿意分數會趨近什麼值？ (10)

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表一、標準常態分配：累積分配函數$\Pr(Z<z)$

不同z值下的對應函數值

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990