501102

國立臺灣科技大學 106 學年度碩士班招生試題

系所組別：工業管理系碩士班甲組

科　　目：作業研究

(總分為 100 分)

1. Consider the following model.

$$\begin{aligned}
\text{Maximize}\quad & Z = 3x_1 + 5x_2 \\
\text{Subject to}\quad & x_1 \le b_1 \\
& 2x_2 \le b_2 \\
& 3x_1 + 2x_2 \le b_3 \\
& x_i \ge 0,\quad i = 1, 2
\end{aligned}$$

where b_1, b_2, and b_3 are uncertain parameters (random variables) in the model. Assuming that these parameters have a normal distribution, the following table gives the original estimate for them.

Parameter	Mean	Standard Deviation
b_1	8	0.5
b_2	12	1
b_3	20	2

Suppose that the minimum acceptable probability that the first, second, and third original constraints will hold is 0.90, 0.95, and 0.99, respectively.

(a) Use probability expressions to write the three chance constraints (10%).

(b) What are the deterministic equivalents of these chance constraints? (10%)

[Hint: $K_{0.9} = -1.28, K_{0.95} = -1.645$, and $K_{0.99} = -2.33$, where $P\{\text{standard normal} > K_\alpha\} = \alpha$.]

2. A company supplies water to four cities from three rivers. The supply of each river, the minimum demand and maximum demand of each city, the cost per unit of supplying water from each river to each city are given in the following table.

		City 1	City 2	City 3	City 4	Supply (unit)
River	1	12	8	13	15	120
River	2	6	14	10	9	100
River	3	5	16	7	11	80
Minimum demand (unit)		110	60	0	80	
Maximum demand (unit)		150	60	75	∞	

All the water from the three rivers must be supplied to the cities. The amount of water supplied to each city must be between the city's minimum demand and maximum demand. The company wishes to know how many units of water to supply from each of the rivers to each of the cities to minimize the total cost.

(a) Formulate this problem as a transportation problem by constructing the appropriate cost and requirements table. (10%)

(b) Find a basic feasible solution to the transportation problem formulated in part (a) by Vogel's approximation method. (10%)

(c) Starting with the basic feasible solution obtained in part (b), use the transportation simplex method to find an optimal solution to this problem. (10%)

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3. If $f(x_1, x_2, \ldots, x_n)$ is a convex function on a convex set S, show that for $c \geq 0$, $g(x_1, x_2, \ldots, x_n) = cf(x_1, x_2, \ldots, x_n)$ is a convex function on S, and for $c \leq 0$, $g(x_1, x_2, \ldots, x_n) = cf(x_1, x_2, \ldots, x_n)$ is a concave function on S. (10%)

4. A professor continually gives exams to her students. She can give three possible types of exams, and her class is graded as either having done well or badly. Let p_i denote the probability that the class does well on a type i exam, and suppose that $p_1 = 0.3$, $p_2 = 0.6$, and $p_3 = 0.9$. If the class does well on an exam, then the next exam is equally likely to be any of the three types. If the class does badly, then the next exam is always type 1. What proportion of exams are type i, $i = 1, 2, 3$? (20%)

5. A service center consists of two servers, each working at an exponential rate of two services per hour. If customers arrive at a Poisson rate of three per hour, then, assuming a system capacity of at most three customers,

   (a) What fraction of potential customers enter the system? (10 %)

   (b) What would the value of (a) be if there was only a single server, and his rate was twice as fast (that is, $\mu = 4$)? (10 %)