國立臺灣科技大學106學年度碩士班招生試題

系所組別：機械工程系碩士班甲組、乙組、丙組、丁組

科　　目：工程數學

(總分為100分)

1. Solve the following **first order** ordinary differential equations:

   (a). $\frac{dy}{dx} = \frac{x+y}{x-y}$ (15%)

   (b). $\frac{dy}{dx} = \frac{2(x^3+xy)}{x^2-y}$ (5%)

2. Use the **method of *Laplace transform*** to solve the following initial value problem:

   $$t\ddot{y} + (t-1)\dot{y} + y = 0,\ y(0) = \dot{y}(0) = 0,\ y(1) = 1$$

   What will happen if $\dot{y}(0) \neq 0$? Why? (20%)

3. If $A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & -1 \\ -1 & 1 & 0 \end{bmatrix}$, solve the following **linear** system of ***homogeneous*** ordinary differential equations:

   $$\frac{d}{dt}\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = A\begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$$

   subjected to the following initial conditions:

   $$\begin{bmatrix} y_1(0) \\ y_2(0) \\ y_3(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$ (20%)

4. By evaluating all integrals involved, verify the ***divergence theorem*** for the case that the vector field $\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$, and the volume R bounded by the cone $x^2 + y^2 - (z-1)^2 = 0$ and the plane $z = 2$. (20%)

5. Solve in detail the **boundary value problem**,

   $$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0,\quad 1 \le r \le 2,\quad 0 \le \theta \le \frac{\pi}{2},$$

   with $u(r,0) = u\left(r,\frac{\pi}{2}\right) = 0,\ u(1,\theta) = \sin 2\theta,\ u(2,\theta) = 0.$ (20%)