國立臺北科技大學 106 學年度碩士班招生考試

系所組別：2131 電機工程系碩士班丙組

第一節　工程數學　試題　（選考）

第一頁　共一頁

注意事項：

1. 本試題共六題，共 100 分。
2. 請標明大題、子題編號作答，不必抄題。
3. 全部答案均須在答案卷之答案欄內作答，否則不予計分。

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1. Consider the differential equation: $(\cos(x) - 2xy) + (e^y - x^a)y' = 0$.

   (a) (5%) Determine a so that the equation is exact.
   (b) (10%) Find the general solution of the exact solution.

2. (15%) Consider the differential equation: $x^2 y'' - 7xy' + 16y = 0$ for $x > 0$, with one given solution $y_1 = x^4$. Find the second independent solution y_2 by using the technique of reduction of order.

3. (20%) Solve the differential equation: $y''(t) + 5ty'(t) - 5y(t) = 0$ with zero initial conditions $y(0) = 0;\ y'(0) = 10$. (Assume that $\lim\limits_{s \to \infty} Y(s) = 0$)

4. (15%) Consider the matrix $M = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$. Find the eigenvalues of M.

5. Consider that matrix $A^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 3 & -2 & 0 & 2 & 3 \\ 1 & -1 & 1 & 2 & 1 \end{bmatrix}$.

   (a) (10%) Find the basis of the range space of A (denoted by $R(A)$) and the basis of the null space of A^T (denoted by $N(A^T)$), respectively.

   (b) (10%) Show that $x_1 \perp x_2$, for all $x_1 \in R(A)$ and $x_2 \in N(A^T)$.

6. (15%) Consider the subspace $V = R(A)$. Find the orthogonal projection of x onto V, $proj_V(x)$, where $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}$ and $x = \begin{bmatrix} 2 \\ 1 \\ 4 \\ 6 \end{bmatrix}$.