



- (1) Consider the 1st order O.D.E.

$$y + 4xy' = 0$$

Please find

- (a) the general solution by using separable method (10%);
(b) the integrating factor of the O.D.E. (10%);
(c) the unique solution that satisfies the initial condition $y(1) = 2$ (5%)
- (2) If $f(t)$ is a function defined for all $t \geq 0$, the Laplace transform of f is defined by

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$$

For all s such that this integral converges. Please find the Laplace transform of the following functions using the definition above.

- (a) $f(t) = \sin(2t)$ (10%)
(b) $f(t) = 3e^{-t}$ (15%)



(3) (25%)

Please find the tangent plane and normal line to the surface $z = x^2 + y^2$ at the point $(2, -2, 8)$.

(4) (25%)

Given $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$, Please find

- (1) The eigenvalues of A .
- (2) A^N for a given positive integral N .