

# 國立臺灣師範大學 106 學年度碩士班招生考試試題

科目：微積分

適用系所：全球經營與策略研究所

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. List all values of  $x$  for which the given function is not continuous.

(a)  $f(x) = \frac{x^3 + 4x}{(x-2)(2x+3)}$  (5 分)

(b)  $g(x) = \begin{cases} x^3 + 2x - 33 & \text{if } x \leq 3 \\ \frac{x^2 - 6x + 9}{x-3} & \text{if } x > 3 \end{cases}$  (5 分)

2. Tom manages an appliance manufacturing firm. He determines that when blenders are priced at  $p$  dollars apiece, the number sold each month will be

$$D(p) = \frac{8000}{p}$$

Furthermore, he estimates that  $t$  months from now, blenders will be selling at a price of  $p(t) = 0.06t^{3/2} + 22.5$  dollars apiece. At what rate should Tom expect the monthly demand  $D(p)$  to be changing with respect to time 25 months from now? Will the demand be increasing or decreasing at that time? (10 分)

3. The price  $p$  (dollars) of each unit of a particular commodity is estimated to be changing at the rate

$$\frac{dp}{dx} = \frac{-135x}{\sqrt{9+x^2}}$$

where  $x$  (hundred) units are the consumer demand (the number of units purchased at that price). Suppose 400 units ( $x=4$ ) are demanded when the price is \$30 per unit.

- (a) Find the demand function  $p(x)$ . (5 分)  
(b) At what price will 300 units be demanded? (5 分)  
(c) At what price will no units be demanded? (5 分)

4. Evaluate the given definite integral using the fundamental theorem of calculus

(a)  $\int_1^4 (5-2t)dt$  (5 分)      (b)  $\int_{1/3}^{1/2} \frac{e^{1/x}}{x^2} dx$  (5 分)

(c)  $\int_e^{e^2} \frac{1}{x \ln x} dx$  (5 分)      (d)  $\int_0^{\pi/6} \sin^2 t \cos t dt$  (5 分)

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5. The cost of producing  $x$  units of a particular commodity is  $C(x)$  hundred dollars, where

$$C(x) = 450 + 15 \sin \left[ \frac{\pi}{24} (x - 11) \right] \quad \text{for } 0 \leq x \leq 30$$

Find the marginal cost of producing 3 units. (5 分)

6. A commodity is introduced with an initial price of \$4 per unit, and  $t$  months later, the price is  $p(t)$  dollars per unit. A study indicates that at time  $t$ , the price changes at a rate equal to 2% of the shortage  $D-S$ , where  $S(t)=2+p$  and  $D(t)=3+7e^{-t}$  are, respectively, the supply and demand for the commodity, both in thousands of units.

(a) Set up and solve an initial value problem for the unit price  $p(t)$ . (10 分)

(b) What is the unit price after 6 months? (5 分)

7. The marginal profit derived from producing  $x$  units of a particular commodity is  $P'(x)$  thousand dollars per unit, where

$$P'(x) = 10x^2 e^{-x^2}$$

Find the Taylor polynomial of degree 8 for  $P'(x)$  at  $x=0$  and integrate to obtain an estimate of the net profit  $P(1) - P(0)$  obtained from producing the first unit. (10 分)

8.  $f(x,y)$  is a joint probability density function for the random variables  $X$  and  $Y$ .

$$f(x,y) = \begin{cases} \frac{1}{16}(x^2 + 2y) & 0 \leq x \leq 1, 1 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Use integration to find the probability of  $P(0 \leq x \leq 1, 1 \leq Y \leq 2X + 1)$  (5 分)

9. A manufacturer of television sets make two models, the Deluxe and the Standard. The manager estimates that when  $x$  hundred Deluxe sets and  $y$  hundred Standard sets are produced each year, the annual profit will be  $P(x, y)$  thousand dollars, where

$$P(x, y) = -0.3x^2 - 0.5xy - 0.4y^2 + 85x + 125y - 2,500$$

The company can produce exactly 30,000 sets each year. How many Deluxe and how many Standard sets should be produced each year to maximize annual profit? (10 分)