國立臺灣師範大學106學年度碩士班招生考試試題

科目：工程數學　　適用系所：機電工程學系-光機電系統組

注意：1.本試題共2頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

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1. （10分）Solve the second-order differential equation

$$2y''(x)+4y'(x)+20y(x)=100,$$

with $y(0)=0$ and $y'(0)=0$. Roughly sketch the solution for $y(x)$ and show the approach of $y(x)$ to its steady-state solution.

2. （10分）Using the Fourier series of the function

$$f(t)=\begin{cases}0, & -\pi<t<0,\\ t, & 0\le t<\pi,\end{cases}$$

determine the sum of the series $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$.

3. （10分）If the impulse response of a system is

$$h(t)=\begin{cases}0, & t\le 0,\\ 2e^{-5t}\cos(12t-\pi/2), & t>0,\end{cases}$$

find the transfer function for the system.

4. （10分）Using double integrals, find the volume under the surface

$$x^2+y^2+z=9,\quad x\ge 0,\quad y\ge 0,$$

and between the planes $y=1$ and $x=2$.

5. （10分）Are the following three polynomials linearly independent?

$$x^2-1,\ x^2+x-2,\ x^2+3x+2.$$

6. （15分）Convert the third-order differential equation

$$y'''(x)-5y''(x)-y'(x)+5y(x)=0$$

to a set of first-order differential equations in matrix form. Find the eigenvalues and eigenvectors of the system matrix in this state-space representation.

7. （15 分）Consider the vector space of polynomials

$$f(x) = a_0 + a_1 x + a_2 x^2$$

defined on the interval $[-1,1]$. Find an orthonormal basis for these polynomials from the set $\{1,\ x,\ x^2\}$ using the Gram-Schmidt process. The inner product for the polynomials is

$$\langle f, g \rangle = \int_{-1}^{1} f(x) g(x)\, dx.$$

8. （10 分）Solve the boundary-value problem

$$y''(x) + y(x) = x,\quad 0 \le x \le \frac{\pi}{2},$$

with $y(0) = 2$ and $y(\pi/2) = 1$.

9. （10 分）Consider an RLC circuit with an input voltage $u(t)$ and an output voltage $y(t)$ shown in Fig. 1. Represent this circuit by $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}u(t)$ and $y(t) = \mathbf{C}\mathbf{x}(t)$, in which $\mathbf{x}(t)$ denotes the state vector. Find the transfer function of the circuit from input to output using $\dfrac{Y(s)}{U(s)} = \mathbf{C}[s\mathbf{I} - \mathbf{A}]^{-1}\mathbf{B}$, in which $Y(s)$ and $U(s)$ denote the Laplace transforms of $y(t)$ and $u(t)$, respectively.

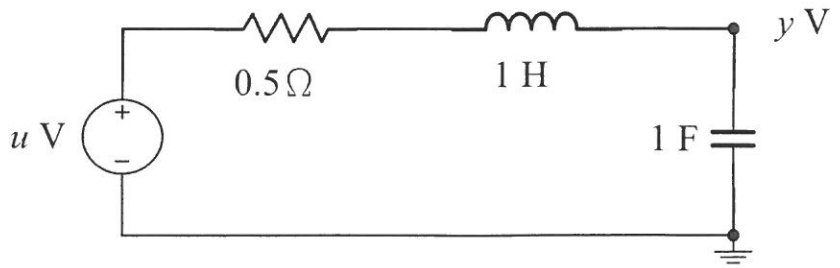

Figure 1. Network for Problem 9