國立臺灣師範大學 106 學年度碩士班招生考試試題

科目：**工程數學**　　　　　　　　　適用系所：**機電工程學系-精密機械組**

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

【試題 1】(15 分)

(1) If the nonhomogeneous linear ODE is $y' + p(x)y = r(x)$, please prove its solution to be

$$y(x) = e^{-h}[\int e^{h} r\, dx + c], \quad h = \int p(x)dx$$

(2) To solve $xy' + 2y = \dfrac{\sin x}{x}$, $y(\dfrac{\pi}{2}) = 1$

【試題 2】(15 分)

Find a general solution by the method of undetermined coefficients.

$$y'_1 = y_1 + y_2 + 5\cos t$$
$$y'_2 = 3y_1 - y_2 - 5\sin t$$

【試題 3】(15 分)

Use the Laplace transform to solve

$$ty'' - 2ty' - 2y = 0, \ \ y(0) = 0, \ \ y'(0) = 4$$

【試題 4】(15 分)

If the matrix A_n has the eigenvalues $\lambda_1, \lambda_2, ...\lambda_n$, find (1) $\sum_{i=1}^{n} \lambda_i$, (2) the values of $\lambda_1, \lambda_2, ...\lambda_n$, and (3) det (A_n).

$$A_n = \begin{bmatrix} x+a & a & \cdots & \cdots & a \\ a & x+a & \cdots & \cdots & a \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ a & a & \cdots & \cdots & x+a \end{bmatrix}_{n\times n}$$

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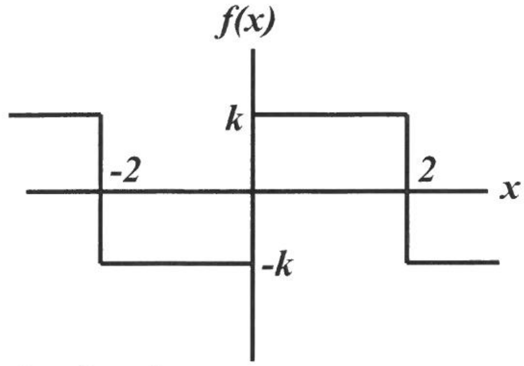
【試題 5】(15 分)

(1) Find the Fourier series of the function

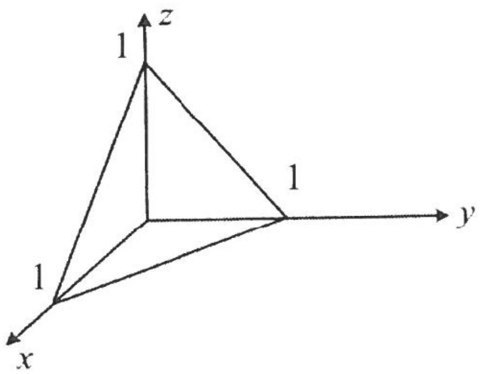
$$f(x)=\begin{cases}-k & \text{if } -2<k<0 \\ k & \text{if } \quad 0<k<2\end{cases}$$



(2) What value is the sum of infinite series: $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots\cdots$

【試題 6】(15 分)

Please evaluate flux integral $\iint_S \vec{F}\bullet\vec{n}dA$ when $\vec{F}=[2x^2,\ 4y,\ 0]$ and S is the portion of the plane $x+y+z=1$ in the first octant $(x\geq 0,\ \ y\geq 0,\ \ z\geq 0)$.



【試題 7】(10 分)

To solve the integral $\oint_C \frac{i\sin z}{2z-\pi}dz$ by Cauchy's integral formula, where C is a simple closed contour enclosing $z_0=\frac{\pi}{2}$