

國立臺灣師範大學 106 學年度碩士班招生考試試題

科目：基礎數學

適用系所：數學系

注意：1.本試題共 2 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

Part I : Calculus

1. (10 分) Use $\epsilon - \delta$ definition of limits to show that $f(x) = x^2 + 1$ is continuous at $x = 1$.

2. (8 分) Find the $\lim_{x \rightarrow 1} \frac{\int_1^{x^2} \frac{1}{t^4 + 1} dt}{\sin \pi x}$.

3. (12 分) Find the integrals

(a) $\int \frac{1}{(1+x^2)^2} dx$ (b) $\int \frac{2x^3 - 8x^2 + 10x + 3}{x^2 - 4x + 5} dx$

4. (10 分) Let $f(x, y) = \begin{cases} \frac{3x^2y}{x^4+y^2}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$

(a) Show that $f_x(0, 0) = f_y(0, 0) = 0$.

(b) Show that f is not continuous at $(0, 0)$.

5. (10 分) Find all relative extremes and saddle points of the function

$$f(x, y) = x^3 - y^3 - 2xy + 6.$$

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Part II : Linear Algebra

6. (12 points) Suppose that A is an invertible $n \times n$ matrix and B is similar to A . Let A^T denote the transpose of A . Mark each of the following True or False.

- (a) The matrices A and B have the same eigenvalues.
- (b) The matrices A and B have the same eigenvectors.
- (c) The matrices A and A^T have the same eigenvalues.
- (d) The matrices A and A^T have the same eigenvectors.
- (e) The matrices A and A^{-1} have the same eigenvalues.
- (f) The matrices A and A^{-1} have the same eigenvectors.

7. Let $A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{bmatrix}$.

- (a) (6 points) Find the eigenvalues of A .
- (b) (10 points) Give a 3×3 matrix B such that $B^{11} = A$ and prove your answer.

8. Let $A = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 2 & -1 & 2 & 0 \end{bmatrix}$. Let W be the nullspace of A and let W^\perp denote the

orthogonal complement of W .

- (a) (8 points) Find an orthonormal basis for W and W^\perp , respectively.
- (b) (6 points) Find the projection of $[2, 0, 1, 7] \in \mathbb{R}^4$ on the subspace W^\perp .
- (c) (8 points) Let T be the reflection of \mathbb{R}^4 through W . Find the standard matrix of T .

(試題結束)