國立臺南大學 106 學年度 **應用數學系碩士班** 招生考試 微積分 試題卷

1. Find the integral $\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx$. (8%)

2. Determine whether the integral $\int_1^4 \frac{1}{(x-2)^2}\,dx$ converges. (8%)

3. Find the limit $\lim_{x\to\infty}\left(\sin\sqrt{x+1}-\sin\sqrt{x}\right)$. (10%)

4. Find the equation for the tangent line to the curve $\left(x^2+y^2\right)^2=(x-y)^2$ at the point $(1,0)$. (10%)

5. Let

$$f(x)=\begin{cases} x^2\sin\frac{1}{x}, & x\neq 0,\\ 0, & x=0.\end{cases} \qquad (14\%)$$

(1) Prove that $f(x)$ is continuous at $x=0$.

(2) Find the derivative of $f(x)$.

6. Test the following series for convergence or divergence: (15%)

(1) $\sum_{n=1}^{\infty}\frac{1}{100+4^n}$.

(2) $\sum_{n=1}^{\infty}\frac{1}{n^{3/5}}$.

(3) $\sum_{n=1}^{\infty}(-1)^n\frac{n^3}{3^n}$.

7. Define $f(x,y)=xe^{-y}+ye^{-x}$. (20%)

(1) Please explain if $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$ exist.

(2) Find the gradient $\nabla f(0,0)$.

(3) Find the maximum rate of change of f at the point $(0,0)$. Also, find the direction in which it occurs (you can describe it in terms of a vector).

8. Suppose that D is the half-annulus given by

$$1\le x^2+y^2\le 4,\ \ y\ge 0.$$

Evaluate (15%)

$$\iint_D \sin(x^2+y^2)\,dA.$$