國立臺灣師範大學 105 學年度碩士班招生考試試題

科目：**工程數學** 適用系所：**光電科技研究所**

注意：1.本試題共 1 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則依規定扣分。

1. Consider a series Resistor-Inductor circuit. The current is satisfied the first-order linear differential equation $L\frac{di}{dt}+Ri=V(t)$. If the initial condition at $t=0$ is $i(0)=i_0$ and the driven voltage is $V(t)$ = constant = V_0, show that the solution for $i(t)$ is $i(t)=\frac{V_0}{R}+\left(i_0-\frac{V_0}{R}\right)e^{-Rt/L}$. (10 分)

2. Consider the fourth-order differential equation $y^{(iv)}-8y'''+26y''-40y'+25y=0$, where y is a function of x. Find the general solution. (10 分)

3. The gamma function is defined as $\Gamma(x)=\int_0^{\infty}t^{x-1}e^{-t}dt$, where $x>0$. Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$. (10 分)

4. Consider a series Resistor-Capacitor circuit. The charge on the capacitor is governed by the differential equation, $R\frac{dQ}{dt}+\frac{1}{C}Q=V(t)$. Let $V(t)$ be an impulse voltage, i.e., $V(t)=V_0\delta(t-T)$, and initial charge is $Q(0)=Q_0$. Please use the **Laplace Transform** to find the solution of $Q(t)$. (15 分)

5. Use the **eigenvalue problem method** to solve the coupled differential equations $\begin{cases}x'=x+4y\\ y'=x+y\end{cases}$, where x and y are function s of time t. (15 分)

6. Evaluate the integral $I=\iiint_R yz^2dV$, where R is the closed region bounded by the planes $x=0$, $y=0$, $z=0$, and $3x+2y+6z=6$. Here, $dV=dxdydz$. (15 分)

7. (a) Write down the Green's Theorem in 2-dimensional space for a vector field $\vec{V}=P(x,y)\hat{i}+Q(x,y)\hat{j}$. (b) If $P(x,y)=xy^3$, $Q(x,y)=x^2-y^2$, please verify the Green's theorem over a region enclosed by $y=0$, $x=1$, and $y=2x$. (15 分)

8. Write down the (a) Stokes' s Theorem, and (b) Divergence Theorem, in 3-dimensional space. (10 分)