**Part I**: **選擇題 (單選題, 每題 5 分, 不需計算過程)**: you are allowed to use your calculator:

1. Chebyshev's Theorem states:
   a) At least $(1-\frac{1}{z^2})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 1.
   b) At least $(1-\frac{1}{z^2})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 0.
   c) At least $(1-\frac{\pi}{z^2})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 1.
   d) At least $(1-\frac{\pi}{z})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 1.
   e) At least $(1-\frac{\pi}{z^2})$ of the data values must be within z standard deviations of the mean, where z is any value greater than 0.
2. You have the following numbers: 1,2,3,4,5,6,7. What is the geometric mean?

   a) 3　b) 4　c) 5　d) 3.5　e) 3.38
3. Suppose the Taiwan lottery uses a random selection of 6 integers from a group of 53 to determine the weekly winner. An individual who buys a lottery ticket has 1 chance in x of winning. What is x?

   a) 53　b) 23,123,491　c) 22,957,480　d) 6　e) 232,344,910
4. The prior probabilities for events $A_1$ and $A_2$ are $P(A_1) = 0.40$ and $P(A_2) = 0.60$. It is also known that $P(A_1 \cap A_2) = 0$. Suppose $P(B|A_1) = 0.20$ and $P(B|A_2) = 0.05$. What is $P(B)$?

   a) 0.08　b) 0.09　c) 0.10　d) 0.11　e) 0.12
5. A sample of 15 consumers provided the following product ratings for three different products. Five consumers were randomly assigned to test and rate each product. What is the value of the Kruskal-Wallis test statistic that can be used to determine whether there is a significant difference among the ratings for the products?

| Product | | |
|---|---|---|
| A | B | C |
| 50 | 80 | 60 |
| 62 | 95 | 45 |
| 75 | 98 | 30 |
| 48 | 87 | 58 |
| 65 | 90 | 57 |

   a) 5.32　b) 6.43　c) 7.89　d) 9.53　e) 10.22

6. Consider the following time series data. Use Holt's linear exponential smoothing method with $\alpha = 0.3$ and $\beta = 0.5$ to develop a forecast for $t = 6$.

| t | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $Y_t$ | 6 | 11 | 9 | 14 | 15 |

a) 21.16
b) 17.17
c) 16.23
d) 15.58
e) 16.05

7. Consider the following time series data. What is the three-week moving average forecast for week 7?

| Week | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Value | 18 | 13 | 16 | 11 | 17 | 14 |

a) 16
b) 11
c) 14
d) 15
e) 13

8. Suppose the loading time for a truck follows the exponential probability distribution. The mean loading time is 15 minutes. What is the probability that loading a truck will take 6 minutes or less?
a) 1.2538
b) 1.0000
c) 0.9335
d) 0.6775
e) 0.3297

9. Apple computers stock provides the following returns for the previous week. What is the variance of Apple computers stock returns for the previous week?

| | |
|---|---|
| Monday | 0.01 |
| Tuesday | 0.01 |
| Wednesday | 0.02 |
| Thursday | 0.01 |
| Friday | 0.02 |

a) .32323
b) .03232
c) .00282
d) .13131
e) .00003

**Part II. 計算題（需詳列計算過程）**

10. (10 points) There is a bivariate distribution for the random variables X and Y as follows.

| f(x, y) | x | y |
|---|---|---|
| 0.2 | 50 | 80 |
| 0.5 | 30 | 50 |
| 0.3 | 40 | 60 |

(a) Compute the variance for $2X + 3Y$.
(b) Compute the covariance and correlation for X and Y. How are they related (positively, negatively, or unrelated)?

11. (15 points) There is a population $\Pi = \{2, 3, 3\}$. Take possible samples of size 2, denoted by $(X_1, X_2)$, and consider the sample mean $\bar{X} = \frac{X_1 + X_2}{2}$.
(a) If sampling without replacement, construct the probability distribution for $\bar{X}$.
(b) Using (a), compute the expected mean and the variance of the sample mean, that is, $E(\bar{X})$ and $Var(\bar{X})$.
(c) If sampling with replacement, also compute $E(\bar{X})$ and $Var(\bar{X})$.

12. (5 points) Consider $\{X_i\}_{i=1}^{30}$ which independently and identically follow a uniform distribution U(0,1). According to Central Limit Theorem, find $P(\sum_{i=1}^{30} X_i \leq 18)$.

13. (5 points) Take two independent samples from two populations as follows.

| | | |
|---|---|---|
| Sample 1 | $n_1 = 200$ | $\bar{p}_1 = 0.22$ |
| Sample 2 | $n_2 = 300$ | $\bar{p}_2 = 0.16$ |

Consider a hypothesis test using $\alpha = 0.05$:

$$H_0: p_1 - p_2 \leq 0$$
$$H_1: p_1 - p_2 > 0$$

What is your testing conclusion?

14. (20 points) Data on years of working experience and monthly sales (in thousands of dollars) for a sample of 7 salespersons in a company as follows.

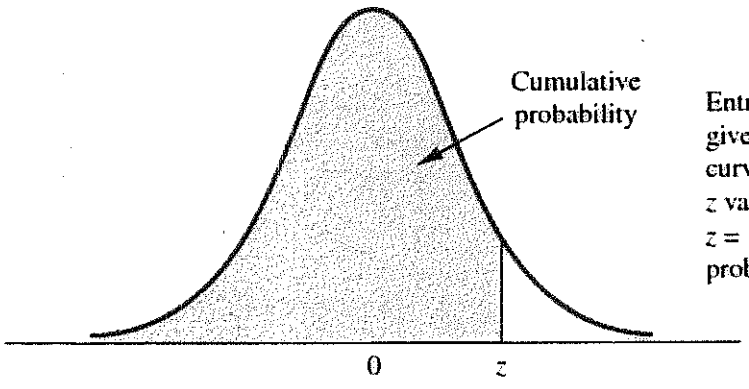
| years of working experience | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| monthly sales | 3 | 5 | 8 | 10 | 10 | 12 | 15 |

Let X denote years of working experience and Y denote monthly sales. Use the data to fit a simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$.

(a) Develop the estimated simple linear regression equation $\hat{y}_i = b_0 + b_1 x_i$.
(b) Compute the coefficient of determination $r^2$.
(c) Test the following hypothesis ($\alpha = 0.05$):

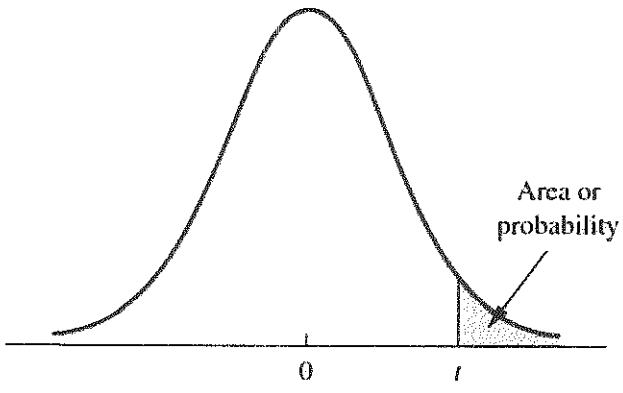
$$H_0: \beta_1 = 0$$
$$H_1: \beta_1 \neq 0$$

(d) Develop a 95% confidence interval for the expected value of $y$ when $x = 4$.




Entries in the table give the area under the curve to the left of the $z$ value. For example, for $z = 1.25$, the cumulative probability is .8944.

| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |




Entries in the table give $t$ values for an area or probability in the upper tail of the $t$ distribution. For example, with 10 degrees of freedom and a .05 area in the upper tail, $t_{.05} = 1.812$.

| Degrees of Freedom | Area in Upper Tail | | | | | |
|---|---|---|---|---|---|---|
| | .20 | .10 | .05 | .025 | .01 | .005 |
| 1 | 1.376 | 3.078 | 6.314 | 12.706 | 31.821 | 63.656 |
| 2 | 1.061 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 |
| 3 | .978 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 |
| 4 | .941 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 |
| 5 | .920 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 |
| 6 | .906 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 |
| 7 | .896 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 |
| 8 | .889 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 |
| 9 | .883 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 |
| 10 | .879 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 |
| 11 | .876 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 |
| 12 | .873 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 |
| 13 | .870 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 |
| 14 | .868 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 |
| 15 | .866 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 |
| 16 | .865 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 |
| 17 | .863 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 |
| 18 | .862 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 |
| 19 | .861 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 |
| 20 | .860 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 |
| 21 | .859 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 |
| 22 | .858 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 |
| 23 | .858 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 |
| 24 | .857 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 |
| 25 | .856 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 |
| 26 | .856 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 |
| 27 | .855 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 |
| 28 | .855 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 |
| 29 | .854 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 |