一、計算題(共80分)：

1. (20%) Let $x(t)=\cos(2\pi f_0 t+\theta)$ be a sinusoidal signal, where f_0 is a fixed number and θ is an arbitrary number. Consider the truncated $x_T(t)=\Pi\left(\frac{t}{T}\right)x(t)$, where

$$\Pi\left(\frac{t}{T}\right)=\begin{cases}1, & -\frac{T}{2}<t<\frac{T}{2} \\ 0, & \text{otherwise}\end{cases}.$$

   a). (10%) Compute the continuous-time Fourier transform (CTFT) of $x_T(t)$, i.e., $X_T(f)=\mathcal{F}\{x_T(t)\}$. Hint: Use the multiplication property for CTFT.

   b). (5%) Compute the power spectral density by $S_x(f)=\lim_{T\to\infty}\frac{\left|\mathcal{F}\{x_T(t)\}\right|^2}{T}$. Hint:

   $\delta(f)=\lim_{T\to\infty}\frac{1}{T}\left(\frac{\sin(\pi Tf)}{\pi f}\right)^2$.

   c). (5%) Compute the power of $x(t)=\cos(2\pi f_0 t+\theta)$ by taking the integration of the $S_x(f)$ in part (b) over all frequencies.

2. (20%) Let $\Pi(t)=\begin{cases}1, & -\frac{1}{2}<t<\frac{1}{2} \\ 0, & \text{otherwise}\end{cases}$ be the standard rectangular pulse, W is a fixed number, and $f_c \gg W$ is the carrier frequency. Consider $M_1(f)=\Pi\left(\frac{f}{W}\right)$ and

$M_2(f)=\Pi\left(\frac{f-\frac{W}{4}}{\frac{W}{2}}\right)-\Pi\left(\frac{f+\frac{W}{4}}{\frac{W}{2}}\right)$ as the CTFTs of $m_1(t)$ and $m_2(t)$, respectively. Suppose that one tries to transmit both $m_1(t)$ and $m_2(t)$ simultaneously by

$u(t)=m_1(t)\cos(2\pi f_c t)+m_2(t)\sin(2\pi f_c t)$.

   a). (5%) What is the CTFT of $u(t)$?
   b). (5%) Sketch your result in part (a).
   c). (5%) What is the bandwidth occupied by $u(t)$?
   d). (5%) Assume that the transmitted signal is perfectly received. How do you demodulate the received $r(t)=u(t)$ to obtain $m_1(t)$ and $m_2(t)$?

3. (10%) The Bessel function of the first kind $J_n(\beta)$ is defined to be the Fourier series coefficient of periodic signal $e^{j\beta\sin(2\pi f_m t)}$ for fixed β and f_m, i.e.,

$$J_n(\beta) \triangleq f_m \int_{-1/(2f_m)}^{1/(2f_m)} e^{j\beta\sin(2\pi f_m t)} e^{-j2\pi n f_m t} dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-[jnx-\beta\sin(x)]} dx.$$

Show that $\sum_{n=-\infty}^{\infty} |J_n(\beta)|^2 = 1$

4. (10%) The random variables X and Y are independent identically distributed according to $\mathcal{N}(0,\sigma^2)$. Let random variable Z be defined by $Z \triangleq \sqrt{X^2+Y^2}$.

   a). (7%) Compute the cumulative density function $F_Z(z) \triangleq \Pr\{Z \le z\}$.

   b). (3%) Compute the probability density function $f_Z(z)$.

5. (20%) Assume that *equally likely* 4-PAM symbol $s \in \left\{-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right\}$ is transmitted over the additive white Gaussian noise channel. The received real-valued signal is $r = s + w$, where noise w is distributed according to $\mathcal{N}(0,\sigma^2)$. Consider the minimum-distance demodulator $\hat{s} = \arg\min_s |r - s|$.

   a). (5%) Express the conditional error probability given $s = -\frac{3}{2}$, i.e., $\Pr\left\{\hat{s} \ne s \middle| s = -\frac{3}{2}\right\}$, in terms of the Gaussian Q-function. Hint: Gaussian Q-function $Q(x) \triangleq \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2} dt$.

   b). (5%) Express the conditional error probability given $s = -\frac{1}{2}$, i.e., $\Pr\left\{\hat{s} \ne s \middle| s = -\frac{1}{2}\right\}$, in terms of the Gaussian Q-function.

   c). (10%) Express the error probability $\Pr\{\hat{s} \ne s\}$ in terms of the Gaussian Q-function.

二、名詞解釋(20分)：請以下列名詞為標題，利用數學符號、數學式、圖、表格或其他專業術語寫一短文，字數至少200字，從該名詞的定義、用途、特性等各角度，解釋下列的名詞。

1. (20%) Double Sideband Modulation