Linear Algebra

1. (10%) Let R^3 have the Euclidean inner product. The subspace of R^3 spanned by the vectors $\mathbf{u}_1 = (\frac{4}{5}, 0, \frac{-3}{5})$ and $\mathbf{u}_2 = (0,1,0)$ is a plane passing through the origin. Express $\mathbf{w} = (1,2,3)$ in the form $\mathbf{w} = \mathbf{w}_1 + \mathbf{w}_2$, where $\mathbf{w}_1$ lies in the plane and $\mathbf{w}_2$ is perpendicular to the plane.

2. (15%) (a) Show that if $\mathbf{v}$ is any $n\times1$ matrix and I is the $n\times n$ identity matrix, then $I - \mathbf{v}\mathbf{v}^T$ is orthogonally diagonalizable.

   (b) Find a matrix P that orthogonally diagonalizes $I - \mathbf{v}\mathbf{v}^T$ if

$$\mathbf{v} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

3. (5%) Let $A = [a_{ij}]$ be a 2x2 matrix with $a_{22} \neq 0$. The matrix A can be factored into a product of the form

$$\begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & 0 \\ a_{21} & y \end{bmatrix}$$

   What is the values of x?

4. (10%) In coding a message, a blank space was represented by 0, an 'A' by 1, a 'B' by 2, a 'C' by 3, and so on. The message was transformed using the matrix

$$C = \begin{bmatrix} -1 & 1 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

   and sent as

   0, 0, -18, 21, 12, 8, -16, 11

   What was the message?

5. (10%) In $\mathbf{P}_3$, there are two ordered bases $p_1 = [x\text{-}1, x^2\text{-}1]$ and $p_2 = [x^2+2x\text{-}3, 4x^2\text{-}x\text{-}3]$. Find the transition matrix corresponding to the change coordinates from p_1 to p_2.

國立中正大學106學年度碩士班招生考試試題

系所別：電機工程學系-
電磁晶片組
計算機工程組
電力與電能處理甲組
電力與電能處理乙組

科目：線性代數與微分方程

第 2 節

微 分 方 程

1. (10%) Find the solution for the following initial value problem (IVP):

$$4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}+y=0, \quad y(1)=0, y(e)=e^{-1/2}.$$

2. (15%) Find the Fourier series of f on the given interval:

$$f(x)=\begin{cases}1, & -\pi<x<0\\ 0, & 0\le x<\pi.\end{cases}$$

3. (15%) For what value(s) of $\lambda>0$, if any, does the boundary value problem

$$y''+\lambda^2y=\sin 2x, \quad y(0)=0, y(\pi)=0$$

have (a) one solution, (b) no solutions, (c) infinitely many solutions?

4. (10%) For the differential equation

$$y'=xy, \quad y(0)=1,$$

use the improved Euler's method with $h=0.1$ to approximate the solution at $x=0.1$