

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：統計學【經濟所碩士班】

題號：403003

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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Answer the following five questions, equally weighted

請依題序在答案卷上作答 (5 大題, 共 100 分)

1. (20%) Let $f(x, y) = \frac{x+2y}{12}$, $(x, y) \in \{(0, 1), (0, 2), (1, 0), (1, 1), (2, 0)\}$. Let $U = X + Y$. Find the density function of U .

2. (20%) Let X be uniformly distributed on $(-1, 1)$, and let $Y = X^2$. Find $\text{cov}(X, Y)$.

3. (20%) Suppose that X_1, X_2 , and X_3 are independent and that

$$X_1 \sim N(2, 4), \quad X_2 \sim N(3, 25), \quad X_3 \sim N(4, 16).$$

Let $W = X_1 + 2X_2 - 3X_3 - 4$. Find $P(W > 0)$.

4. (20%) Let X_1, \dots, X_n be independent, $n \geq 2$, and $X_i \sim N(\mu, \sigma^2)$ where $\theta = (\mu, \sigma^2)$ is the unknown parameter.

(a). Find the maximum likelihood estimator (MLE), $\hat{\sigma}^2$ for σ^2 .

(b). Is $\hat{\sigma}^2$ a consistent estimator for σ^2 ? Explain.

5. (20%) Let $\mathbf{x} = (X_1, X_2, X_3, X_4)'$ have a four-variate normal distribution with $E(X_1) = 2$, $E(X_2) = 1$, $E(X_3) = 3$, $E(X_4) = 6$, $\text{var}(X_1) = 1$, $\text{var}(X_2) = 2$, $\text{var}(X_3) = 3$, $\text{var}(X_4) = 4$, and $\text{cov}(X_i, X_j) = 1$ for all $i \neq j$.

(a). What is the conditional distribution of $(X_1, X_2)'$, given $(X_3, X_4)'$?

(b). What is the joint distribution of $Y_1 = X_1 + X_2 + X_3$ and $Y_2 = X_1 - 2X_4$?

背面有題

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Standard Normal Probabilities

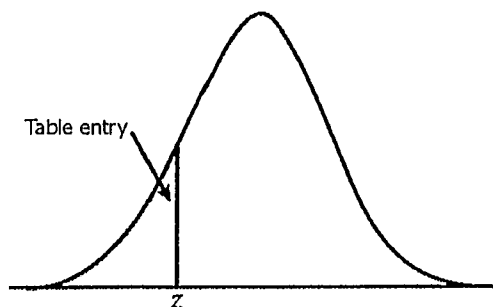


Table entry for z is the area under the standard normal curve to the left of z .

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641