國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【通訊所碩士班甲組】　　題號：437006

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（選擇題）　　

For each of the following questions, please select the best answer from the choices provided. (單選) You do NOT need to provide any justification.

1. (5%) Which of the following statement is **False**?
(A) If an augmented matrix $[\mathbf{A}\ \mathbf{b}]$ is transformed into $[\mathbf{C}\ \mathbf{d}]$ by elementary row operations, then the equations $\mathbf{Ax} = \mathbf{b}$ and $\mathbf{Cx} = \mathbf{d}$ have exactly the same solution sets.
(B) If a system $\mathbf{Ax} = \mathbf{b}$ has more than one solution, then so does the system $\mathbf{Ax} = \mathbf{0}$.
(C) If matrices $\mathbf{A}$ and $\mathbf{B}$ are row equivalent, they have the same reduced echelon form.
(D) If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{Ax} = \mathbf{b}$ is consist for every $\mathbf{b}$ in $\mathbb{R}^m$, then $\mathbf{A}$ has m pivot column.
(E) If $\mathbf{A}$ is an $m \times n$ matrix and the equation $\mathbf{Ax} = \mathbf{b}$ is consistent for some $\mathbf{b}$, then the columns of $\mathbf{A}$ span $\mathbb{R}^m$.

2. (5%) Which of the following statement is **False**?
(A) If $\mathbf{A}$ and $\mathbf{B}$ are row equivalent $m \times n$ matrices and if the columns of $\mathbf{A}$ span $\mathbb{R}^m$, then so do the columns of $\mathbf{B}$.
(B) In some cases, it is possible for four vectors to span $\mathbb{R}^5$.
(C) If $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^m$, then $-\mathbf{u}$ is in Span$\{\mathbf{u}, \mathbf{v}\}$.
(D) If $\mathbf{A}$ is a 6×5 matrix, the linear transformation $\mathbf{x} \mapsto \mathbf{Ax}$ cannot map $\mathbb{R}^5$ onto $\mathbb{R}^6$.
(E) A linear transform is a function.

3. (5%) Which of the following statement is **False**?
(A) If $\mathbf{A}$ and $\mathbf{B}$ are $m \times n$, then both $\mathbf{AB}^T$ and $\mathbf{A}^T\mathbf{B}$ are defined.
(B) Left-multiplying a matrix $\mathbf{B}$ by a diagonal matrix $\mathbf{A}$, with nonzero entries on the diagonal, scales the rows of $\mathbf{B}$.
(C) If $\mathbf{BC} = \mathbf{BD}$, then $\mathbf{C} = \mathbf{D}$.
(D) If $\mathbf{AB} = \mathbf{BA}$ and if $\mathbf{A}$ is invertible, then $\mathbf{A}^{-1}\mathbf{B} = \mathbf{BA}^{-1}$.
(E) An elementary $n \times n$ matrix has either n or $n+1$ nonzero entries.

4. (5%) Which of the following statement is **False**?
(A) If $\mathbf{B}$ is formed by adding to one row of $\mathbf{A}$ a linear combination of other rows, then $\det(\mathbf{A}) = \det(\mathbf{B})$.
(B) $\det(\mathbf{A}^T\mathbf{A}) \geq 0$.
(C) If $\mathbf{A}^3 = \mathbf{0}$, then $\det(\mathbf{A}) = 0$.
(D) $\det(-\mathbf{A}) = -\det(\mathbf{A})$.
(E) If $\mathbf{A}$ is invertible, then $\det(\mathbf{A})\det(\mathbf{A}^{-1}) = 1$.

5. (5%) Which of the following statement is **False**?
(A) If $\mathbf{B}$ is obtained from a matrix $\mathbf{A}$ by several elementary row operations, then $\mathsf{rank}(\mathbf{B}) = \mathsf{rank}(\mathbf{A})$.
(B) Row operations on a matrix $\mathbf{A}$ can change the linear dependence relations among the rows of $\mathbf{A}$.
(C) A change-of-coordinates matrix is always invertible.
(D) If $\mathbf{A}$ is $m \times n$ and linear transformation $\mathbf{x} \mapsto \mathbf{Ax}$ is onto, then $\mathsf{rank}\mathbf{A} = m$.
(E) If $\mathbf{A}$ is $m \times n$ and $\mathsf{rank}\mathbf{A} = m$, then the linear transform $\mathbf{x} \mapsto \mathbf{Ax}$ is one-to-one.

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6. (5%) Which of the following statement is **False**?
   (A) If $\mathbf{A}$ is invertible and 1 is an eigenvalue of $\mathbf{A}$, then 1 is also an eigenvalue of $\mathbf{A}^{-1}$.
   (B) If $\mathbf{A}$ contains a row or column of zeros, then 0 is an eigenvalue of $\mathbf{A}$.
   (C) Each eigenvector of $\mathbf{A}$ is also an eigenvector of $\mathbf{A}^2$.
   (D) Each eigenvalue of $\mathbf{A}$ is also an eigenvalue of $\mathbf{A}^2$.
   (E) Eigenvectors much be nonzero vectors.

7. (5%) Which of the following statement is **False**?
   (A) There exists a 2×2 matrix that has no eigenvectors in $\mathbb{R}^2$.
   (B) If $\mathbf{A}$ is diagonalizable, then the column of $\mathbf{A}$ are linearly independent.
   (C) A nonzero vector cannot correspond to two different eigenvectors of $\mathbf{A}$.
   (D) If $\mathbf{A}$ and $\mathbf{B}$ are invertible $n \times n$ matrices, then $\mathbf{AB}$ is similar to $\mathbf{BA}$.
   (E) If $\mathbf{A}$ is an $n \times n$ diagonalizable matrix, then each vector in $\mathbb{R}^n$ can be written as a linear combination of eigenvectors of $\mathbf{A}$.

8. (5%) Which of the following statement is **False**?
   (A) Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.
   (B) Similar matrices always have exactly the same eigenvalues.
   (C) The matrices $\mathbf{A}$ and $\mathbf{A}^T$ have the same eigenvalues, counting multiplicities.
   (D) Each eigenvector of an invertible matrix $\mathbf{A}$ is also an eigenvector of $\mathbf{A}^{-1}$.
   (E) If $\mathbf{A}$ is similar to a diagonalizable matrix $\mathbf{B}$, then $\mathbf{A}$ is also diagonalizable.

9. (5%) Which of the following statement is **False**?
   (A) If $\mathbf{A}$ is orthogonally diagonalizable, then $\mathbf{A}$ is symmetric.
   (B) If $\mathbf{A}$ is orthogonal matrix, then $\|\mathbf{Ax}\| = \|\mathbf{x}\|$ for all $\mathbf{x}$ in $\mathbb{R}^n$.
   (C) By a suitable change of variable, any quadratic form $\mathbf{x}^T\mathbf{Ax}$ can be changed into one with no cross-product term.
   (D) The largest value of a quadratic form $\mathbf{x}^T\mathbf{Ax}$, for $\|\mathbf{x}\| = 1$, is the largest entry on the diagonal of $\mathbf{A}$.
   (E) If $\mathbf{P}$ is an $n \times n$ orthogonal matrix, then the change of variable $\mathbf{x} = \mathbf{Pu}$ transforms $\mathbf{x}^T\mathbf{Ax}$ into a quadratic form whose matrix is $\mathbf{P}^{-1}\mathbf{AP}$.

10. (5%) Which of the following statement is **False**?
   (A) The set of all vectors in $\mathbb{R}^n$ orthogonal to one fixed vector is a subspace of $\mathbb{R}^n$.
   (B) If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is an orthogonal set and if c_1, c_2, and c_3 are scalars, then $\{c_1\mathbf{v}_1, c_2\mathbf{v}_2, c_3\mathbf{v}_3\}$ is an orthogonal set.
   (C) If a square matrix has orthonormal columns, then it also has orthonormal rows.
   (D) If a vector $\mathbf{y}$ coincides with its orthogonal projection onto a subspace $\mathbf{W}$, then $\mathbf{y}$ is in $\mathbf{W}$.
   (E) If a matrix $\mathbf{U}$ has orthonormal columns, then $\mathbf{UU}^T = \mathbf{I}$.

11. (10%) The dimension of the subspace

$$H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \in \mathbb{R} \right\}$$

is

(A) 1.
(B) 2.
(C) 3.
(D) 4.
(E) 5.

12. (10%) Let

$\mathbf{A} = \begin{bmatrix} .4 & -.3 \\ .4 & 1.2 \end{bmatrix}$. As $k \to \infty$, we obtain $\mathbf{A}^k$

(A) $\begin{bmatrix} -.5 & -1.75 \\ 1.0 & 1.50 \end{bmatrix}$.

(B) $\begin{bmatrix} -.75 & -.5 \\ 1.0 & 1.50 \end{bmatrix}$.

(C) $\begin{bmatrix} -.5 & 1.50 \\ 1.0 & -.75 \end{bmatrix}$.

(D) $\begin{bmatrix} -1.5 & -.75 \\ 1.0 & 2.50 \end{bmatrix}$.

(E) $\begin{bmatrix} -.5 & -.75 \\ 1.0 & 1.50 \end{bmatrix}$.

13. (10%) Let $\mathbf{J}$ be the $n \times n$ matrix of all 1's, and consider $\mathbf{A} = (a - b)\mathbf{I} + b\mathbf{J}$; that is

$$\mathbf{A} = \begin{bmatrix} a & b & b & \dots & b \\ b & a & b & \dots & b \\ b & b & a & \dots & b \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ b & b & b & \dots & a \end{bmatrix}.$$

Then the eigenvalues of $\mathbf{A}$ are

(A) $a + b$, and $a + (n-1)b$.
(B) $a - nb$, and $a + nb$.
(C) $a - b$, and $a + (n-1)b$.
(D) $a - 2b$, and $a + nb$.
(E) $a + b$, and $a - (n-1)b$.

14. (10%) The determinant of

$$\mathbf{A} = \begin{bmatrix} 3a & -7 & 8 & 9 & -6 \\ 0 & 2 & -5 & 7 & 7 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 2 & 4 & -1 \\ 0 & 0 & 0 & -2 & 0 \end{bmatrix}$$

is

(A) -11a.
(B) -12a.
(C) -13a.
(D) -14a.
(E) -15a.

15. (10%) Let $\mathbf{A}$ and $\mathbf{B}$ be 4×4 matrices, with $\det \mathbf{A} = -1$ and $\det \mathbf{B} = 2$. Then, $\det \mathbf{B}^{-1}\mathbf{A}\mathbf{B} + \det \mathbf{A}^T\mathbf{A} + \det 2\mathbf{A} =$

(A) -12.
(B) -14.
(C) -16.
(D) -18.
(E) -20.