國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：機率【通訊所碩士班甲組】　　題號：437005

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（混合題）　　

選擇題(單選，計分方式:不倒扣，答對得該題全部分數，答錯及未作答得零分)

1. (5%) Let $S=\{1,2,3,4\}$ be a sample space of an experiment. If G is the smallest field that contains the sets $\{1\}$ and $\{2,3\}$, which of the following sets is also contained in G?

   (A) $\{3\}$

   (B) $\{4\}$

   (C) $\{1,3\}$

   (D) $\{2,4\}$

   (E) $\{1,3,4\}$

2. (5%) Let X be a random variable with probability mass function given by

$$p_X(x)=\begin{cases} x^2/a, & \text{if } x=-3,-2,-1,0,1,2,3, \\ 0, & \text{otherwise.} \end{cases}$$

   What is the variance of X?

   (A) 28

   (B) 14

   (C) 9

   (D) 7

   (E) 1

3. (5%) Suppose that X and Y are random variables with the same variance σ^2. What is the covariance of $X-Y$ and $X+Y$?

   (A) 0

   (B) 1

   (C) σ^2

   (D) $2\sigma^2$

   (E) σ^4

4. (5%) Let X be a random variable. Let $M_X(s)$ be the moment generating function associated with X. Which of the following expressions cannot be $M_X(s)$?

   (A) $M_X(s)=\dfrac{e^s}{3-2e^s}$

   (B) $M_X(s)=\dfrac{1}{4}e^{-s}+\dfrac{1}{2}+\dfrac{1}{8}e^{4s}+\dfrac{1}{8}e^{5s}$

   (C) $M_X(s)=\dfrac{1}{3}\cdot\dfrac{3}{2-s}+\dfrac{2}{3}\cdot\dfrac{2}{3-s}$

   (D) $M_X(s)=\dfrac{e^{3s}}{1-2s}$

   (E) $M_X(s)=\dfrac{2}{2-s}e^{3(e^s-1)}$

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5. (5%) A six-sided die is rolled three times independently. What is the probability that we obtain a sum of 12?
   (A) 1/6
   (B) 1/18
   (C) 1/36
   (D) 1/216
   (E) 25/216.

6. (5%) Two coins are tossed simultaneously. If one of them turned head, what is the probability that the other one also turn head?
   (A) 0.1
   (B) 0.25
   (C) 0.5
   (D) 0.75
   (E) None of these

7. (5%) If the pdf of a continuous random variable is given as
$$f(x) = \begin{cases} x/2, & 0 \le x \le 2, \\ 0, & \text{otherwise}, \end{cases}$$
   what is the value of $P(X = 1)$?
   (A) 0
   (B) 0.25
   (C) 0.5
   (D) 1
   (E) None of these

8. (5%) If a continuous random variable X has the pdf
$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}, \quad -\infty < x < \infty,$$
   what is its mean?
   (A) 0
   (B) 1
   (C) π
   (D) $ln\ (1 + \pi)$
   (E) None of these

9. (5%) If a continuous random variable X has the pdf
$$f(x) = \frac{1}{2} e^{-\frac{x}{2}}, \qquad x \ge 0.$$
   Which of the following statements is wrong?
   (A) $P(0 \le x \le 2) = e^{-1}$
   (B) $\mathbf{E}[X] = 2$
   (C) $\mathbf{Var}[X] = 8$
   (D) The CDF of X is
$$F(x) = 1 - e^{-\frac{x}{2}}, \qquad x \ge 0.$$
   (E) None of these

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10. (5%) Consider two i.i.d. random variables X_1 and X_2 with Poisson distribution:

$$P(X_i = k) = \frac{\lambda^k e^{-\lambda}}{k!}, \qquad k = 0,1,2,3, \ldots\ldots$$

Which of the following statements is wrong?

(A) The mean of X_i is λ

(B) The variance of X_i is λ

(C) The moment generating function of X_i is $e^{\lambda(e^t-1)}$

(D) $X_1 + X_2$ is Poisson distributed

(E) None of these

問答計算題:

1. (10%) Consider two discrete random variables X and Y with joint pmf:

$P(x,y)$	$X=-1$	$X=0$	$X=1$
$Y=2$	0.15	0.15	0.1
$Y=4$	0.05	0.1	0.15
$Y=6$	0.1	0.15	0.05

(a) (5%) Are X and Y independent? Prove it or disprove it.

(b) (5%) Are X and Y uncorrelated? Prove it or disprove it.

2. (15%) Consider a random variable X with pmf:

$$P(X=x) = \begin{cases} 0.4, & x = \pm 1, \\ 0.2, & x = 0. \end{cases}$$

Given $X = x$, the conditional distribution of a random variable Y is Gaussian with $N(x,1)$.

(a) (5%) Find the marginal distribution of Y.

(b) (5%) Find the conditional probability $P(X=1|Y=1)$.

(c) (5%) Find the conditional mean $\mathbf{E}[X|Y=1]$

3. (10%) An exponential random variable has a PDF of the form

$$f_Z(z) = \begin{cases} \lambda e^{-\lambda z}, & \text{if } z \geq 0, \\ 0, & \text{otherwise,} \end{cases}$$

where λ is a positive parameter. Suppose that X and Y are independent exponential random variables with common parameter λ. Please find $E[\max(2X,Y)]$.

4. (15%) Let X be a random variable that takes nonnegative integer values, and is associated with a moment generating function of the form

$$M_X(s) = \frac{2}{9} \cdot \frac{3+4e^{2s}+2e^{3s}}{3-e^s}.$$

(a) (5%) Find $E[X]$.

(b) (10%) Find $E[X \mid X \neq 0]$.