

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【資工系碩士班乙組】

題號：434002

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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1. (16%) If  $A$  is an  $n \times n$  matrix, then  $A$  is called **idempotent** if  $A^2 = A$ . Let  $A$  and  $B$  be  $n \times n$  idempotent matrices.
- 1.1 (4%) Show that  $AB$  is idempotent if  $AB = BA$ .
- 1.2 (4%) Show that if  $A$  is idempotent, then  $A^T$  is idempotent.
- 1.3 (4%) Is  $A+B$  idempotent? Justify your answer.
- 1.4 (4%) Find all values of  $k$  for which  $kA$  is also idempotent.

2. (16%) A periodic signal  $x(t)$  with a period  $T_0 = 10$ ,  $0 \leq t \leq 10$  by the equation  $x(t) = \begin{cases} 0 & 0 \leq t \leq 5 \\ 2 & 5 < t \leq 10 \end{cases}$

- 2.1 (4%) Sketch the periodic function  $x(t)$  over the time interval  $-10 \leq t \leq 20$ .
- 2.2 (4%) Determine the DC coefficient of the Fourier series,  $a_0$ .
- 2.3 (4%) Use the Fourier analysis integral to find  $a_1$ , the first Fourier series coefficient.
- 2.4 (4%) If we add a constant value of one to  $x(t)$ , we obtain the signal  $y(t) = 1 + x(t)$  with  $y(t)$  given over one period by  $y(t) = \begin{cases} 1 & 0 \leq t \leq 5 \\ 3 & 5 < t \leq 10 \end{cases}$ .

The signal can be represented by a Fourier series, but with different coefficients:  $y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$ .

Explain how  $b_0$  and  $b_1$  are related to  $a_0$  and  $a_1$ . Note: you should not have to evaluate any new integrals explicitly to answer this question.

3. (18%) The matrix is  $A = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 2 & -2 \\ 3 & 1 & 1 \end{bmatrix}$

- 3.1 (3%) Find the characteristic polynomial.
- 3.2 (6%) Find the eigenvalues,
- 3.3 (9%) And Find the associated eigenvectors.

4. (20%) Using Laplace transform and showing the details of your work, solve the initial value problem.

$$y_1' = -2y_1 + 3y_2, \quad y_2' = 4y_1 - y_2, \quad y_1(0) = 4, \quad y_2(0) = 3,$$

5. (14%) Find the general solution of the following differential equation.

$$y'' + 4y' + 4y = e^{-2x} \sin 2x$$

6. (16%) Solve the following initial value problem.

$$y''' - 2y'' + 4y' - 8y = 0, \quad y(0) = -1, \quad y'(0) = 30, \quad y''(0) = 28,$$