

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：工程數學【機電系碩士班乙組、丙組】

題號：438002

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）

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1. (20%) Solve the following ODEs.

(a) $y' = (-2x + y)^2 - 7$, $y(0) = 0$. (10%)

(b) $y'' - 2y' + y = 35x^{3/2}e^x$. (10%)

2. (15%) Solve $y'' + y = \begin{cases} 2t, & 0 < t < \pi/2 \\ 0, & t > \pi/2 \end{cases}$ with $\begin{cases} y(\pi/4) = \pi/2 \\ y'(\pi/4) = 2 - \sqrt{2} \end{cases}$ by using the Laplace transform.

3. (15 %) Tank T_1 in Fig. 1 initially contains 300 gal of water in which 150 lb of salt are dissolved. Tank T_2 initially contains 200 gal of pure water. Liquid is pumped through the system as indicated, and the mixtures are kept uniform by stirring. Find the amounts of salt $y_1(t)$ and $y_2(t)$ in T_1 and T_2 , respectively.

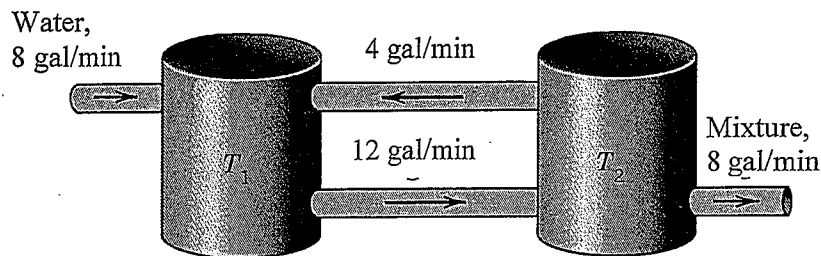


Fig. 1

4. (15%) Multiple choice problem. Suppose \mathbf{A} is a $n \times n$ matrix with rank n . In the following statements, which ones are true?

- (a) \mathbf{A} is a singular matrix.
- (b) $\det(\mathbf{A}) = 0$.
- (c) The nullity of \mathbf{A} is n .
- (d) The column rank of \mathbf{A} is n .
- (e) $\mathbf{Ax} = \mathbf{0}$ has only a trivial solution $\mathbf{x} = \mathbf{0}$.
- (f) $\mathbf{Ax} = \mathbf{b}$ has a unique solution $\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$.
- (g) The column vectors are linearly dependent.
- (h) The row vectors of \mathbf{A} span \mathbb{R}^n .
- (i) \mathbf{A} is diagonalizable.

5. (15%) Compute the flux of a liquid through the surface $S: x^2 + y^2 = 4$, $|z| \leq 2$, where the velocity field is $\mathbf{F} = [\sin(x), \cos(x), \cos(z)]$. (Hint: Divergence theorem of Gauss)

6. (20%) The vibrating string can be modeled by one-dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$. Consider a plastic string of length $L = \pi$ with fixed ends and $c^2 = 1$. It initially has zero displacement and the following velocity:

$$u_t(x, 0) = \begin{cases} 0.01x, & \text{if } 0 \leq x \leq \frac{1}{2}\pi \\ 0.01(\pi - x), & \text{if } \frac{1}{2}\pi \leq x \leq \pi \end{cases}$$

Find the displacement $u(x, t)$ of the string.