國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：離散數學【電機系碩士班丙組】　　　　　　　　　　　　　　　題號：431011

※本科目依簡章規定「可以」使用計算機（廠牌、功能不拘）（問答申論題）　　共2頁第1頁

（答題需將推導過程與原因寫出，回答到要點原因與推導的嚴謹性為主要評分考量）

1. (10%) Given Boolean variables p, q, and r where $\neg p$ is the complement of p, $\wedge$ is logical AND, $\vee$ is logical OR, $\rightarrow$ is logical imply, and $\Leftrightarrow$ is equivalence, prove that the following formula is true.

$$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$$

2. (15%) Given a directed graph G as in Figure 1, derive a relation matrix R of the graph, and derive the matrix of transitive closure R^* of relation R.

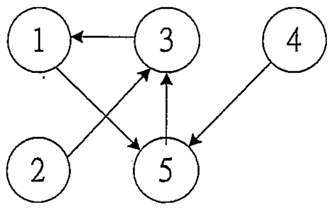

Figure 1

3. (15%) In a 2D n x n checkerboard as shown in Figure 2, given two integers p and q ($1 \le p \le q \le n$), write the formula of the number of all possible squares in the checkerboard with side length s satisfying $p \le s \le q$. Write the result in a formula of n, p, and q.

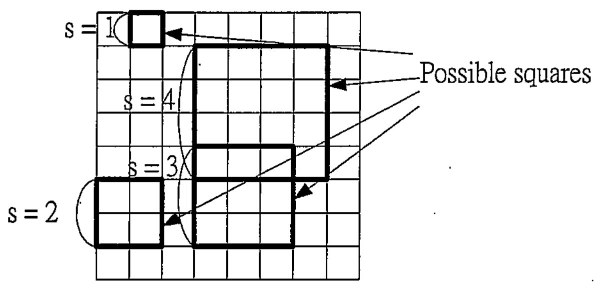

Figure 2

4. (15%) Given a bag having 4 white balls and 3 red balls, let us proceed a drawing run that a drawing takes a ball from the bag, record its color, put it back, and do such drawing for 5 times. Assume that the probability of each distinct red ball being drawn is two times of that of a distinct white ball being drawn. Calculate the probability to draw exactly 3 white balls and 2 red balls in the run.

5. (10%) A complete graph K_n is a graph $G(V,E)$ with n vertices ($n \ge 1$) that have an edge between each pair of vertices in the vertex set. Write the formula of the number of **all** complete subgraphs K_p existing in K_n. ($1 \le p \le n$) The formula is a function of n.

6. (10%) Write an algorithm *queue_partitioning(q, k)* to reorder all data elements in a queue q. Assume that the queue q has n integer elements initially. Given an integer $k > 1$, the reordering will form k ordered partitions of these integers. The resulting queue should be in the order of partition 0, partition 1, ... , and partition k-1. The i^{th} partition of these integer elements contains all integer elements e in the original queue such that e mod $k = i$. (In this algorithm, you can utilize an empty queue $q2$ as a local variable.) You can use a function *length*(q) to get the length of the queue q.

For illustration, Figure 3 shows an example with an initial queue state, $k = 3$, ordered partitions, and a final queue state.

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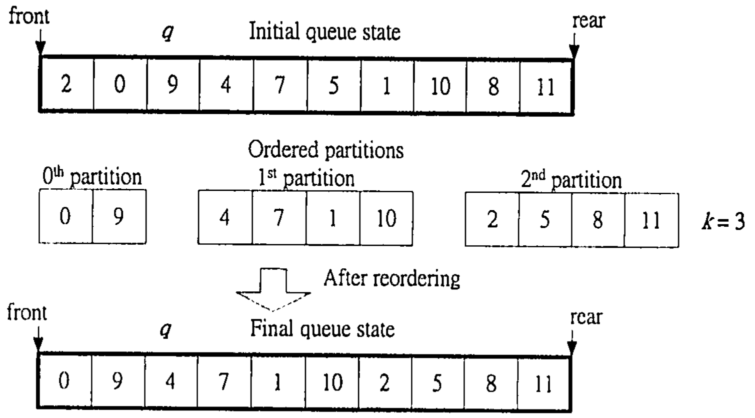

Figure 3

7. (15%) A binary expression tree data structure uses the node data element given as follows. A node either represents a value or represents an arithmetic operator in { +, –, *, / }. An evaluation of an example expression tree is shown in Figure 4. <u>Write a recursive algorithm *evaluate(root)*</u> to perform such expression evaluation task where *root* points to the root element of the expression tree.

```
struct node {
        int    specifier;      // 0:  the node represents a number in the value field
                               // 1 – 4:  the node represents an operator:
                               //              1: + (add), 2: – (subtract), 3: * (multiply), 4: / (divide)
        int    value;
        struct node *   left;     // pointer to the root node of the left subtree
        struct node *   right;    // pointer to the root node of the right subtree
}
```

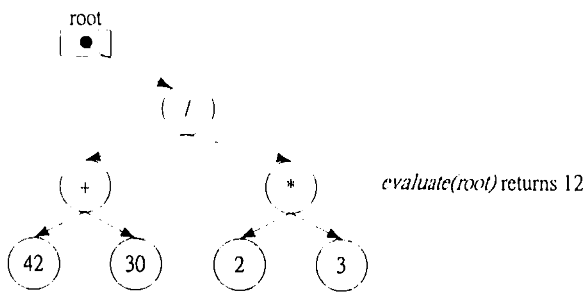

Figure 4

8. (10%) In an undirected graph $G(V, E)$ with n vertices, these is an integer data d_i in each vertex v_i. Given a starting vertex S in the vertex set V, <u>write an algorithm</u> that computes the sum of associated data d_i's of all reachable vertices in V from S. (i.e. A reachable vertex T from the vertex S is a vertex that has a path from S to T.)