

國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：高等微積分【應數系碩士班丙組】

題號：424004

※本科目依簡章規定「不可以」使用計算機(問答申論題)

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答題時請標示題號並詳列計算及推導過程。每小題 10 分。

1. Let $a_1 = 2$ and $a_{n+1} = \frac{a_n}{2} + \frac{1}{a_n}$ for $n = 1, 2, 3, \dots$

(a) Show that $\{a_n\}$ is a decreasing sequence.

(b) Does $\{a_n\}$ converge? If yes, find the limit.

2. Let $f(x) = x^2 \sin(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$.

(a) Is f uniformly continuous on $(-1, 1)$?

(b) Is f differentiable on $(-1, 1)$?

3. Let $f(x) = e^{-\frac{1}{x^2}}$ for $x \neq 0$ and $f(0) = 0$.

(a) Is f differentiable at $x = 0$?

(b) Is f analytic at $x = 0$?

4. Let A and B be compact subsets of a metric space (X, d) .

(a) Is $A \cap B$ compact? Why?

(b) Is $A \cup B$ compact? Why?

5. (a) Find the absolute maximum and minimum values of

$f(x, y) = x^2 - 2x + y^2 - 2y + 8$ on the disk $x^2 + y^2 \leq 9$.

(b) Evaluate the line integral $\int_C (x^3 dx + y dy)$ where C is the curve $y = x^2 + 1$ from $(0, 1)$ to $(2, 5)$.

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