國立中山大學 106 學年度碩士暨碩士專班招生考試試題

科目名稱：線性代數【應數系碩士班丙組】　　　　題號：424003

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　　　共1頁第1頁

共六道題。答題時，每題須寫下題號與詳細步驟。 請依題號順序作答，不會作答題目請寫下題號並留空白。

1. [10%] Let $\{v_1, v_2, v_3\}$ be a linearly independent set in some vector space. Show that the set $\{v_1 + v_2, v_2 + v_3, v_3 + v_1\}$ is also linearly independent.

2. [30%] For any $n \in \mathbb{N}$, denote by $P_n(\mathbb{R})$ the collection of polynomials over $\mathbb{R}$ with degrees less or equal to n. Define a map $T : P_3(\mathbb{R}) \to \mathbb{R}^3$ as

$$T(p(x)) = (p(-1), p(0), p(1))$$

for $p \in P_3(\mathbb{R})$.

(1) [10%] Show that $P_3(\mathbb{R})$ is a vector space.

(2) [10%] Show that T is linear.

(3) [10%] Find bases for the kernel $N(T)$ and range $R(T)$ of T, respectively.

3. [15%] Let T be the linear transform on the set $M_n(\mathbb{R})$ of $n \times n$ matrices over $\mathbb{R}$ defined as $T(A) = A^t$, the transpose of $A \in M_n(\mathbb{R})$. Find all eigenvalues of T.

4. [15%] Find the Jordan form of the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

5. [15%] Let A be an $m \times n$ matrix with rank m. Prove that there exists an $n \times m$ matrix B so that $AB = I_m$, the $m \times m$ identity matrix.

6. [15%] Let V be a vector space with dimension n. Suppose that $T : V \to V$ is a linear transform and that there exists some vector $v \in V$ satisfying $T^{n-1}v \neq 0$ and $T^n v = 0$. Show that T admits the matrix representation

$$\begin{pmatrix} 0 & 1 & & \\ & 0 & \ddots & \\ & & \ddots & 1 \\ & & & 0 \end{pmatrix}$$

with respect to some basis of V.

End of Paper