Please write down all the detail of your computation and solution.

1. (15%) Find a, b, c, d such that the linear system

$$\begin{cases} x + y + 2z = b \\ 2x + ay - 2z = c \\ 3x + 6y + 3z = d \end{cases}$$

   has (1) a unique solution, (2) infinitely many solutions, (3) no solution.

2. (15%) Consider the planar transformation F reflects a vector about the line $x + y = 0$, and then rotates it $30°$ about the origin. Find the inverse transformation of F.

3. (15%) Find the projection matrix onto the plane $x - y + z = 0$ in $\mathbf{R}^3$.

4. (15%) Let A be an $m \times n$ real matrix and $\mathbf{b}$ be an n dimensional real column vector. Show that exactly one of the following statements holds: (1) $A\mathbf{x} = \mathbf{b}$ has a solution $\mathbf{x}$, (2) $A^t\mathbf{y} = \mathbf{0}$ has a solution $\mathbf{y}$ such that $\mathbf{y}^t\mathbf{b} \neq 0$.

5. (20%) Let A be an $n \times n$ matrix. Prove A is diagonalizable if and only if A has n linearly independent eigenvectors.

6. (20%) Let A be an $n \times n$ nonsingular matrix and B be an $n \times m$ matrix. State the fastest numerical method to compute (1) determinant of A, (2) $A^{-1}B$. How many arithmetic operations are needed in (1), and what is the minimal memory needed in (2)?