國立中山大學106學年度碩士暨碩士專班招生考試試題

科目名稱：微積分【應數系碩士班乙組】　　題號：424002

※本科目依簡章規定「不可以」使用計算機(問答申論題)　　

計算題：共7題，子題分數平均分配。答題時，每題都必須寫下題號與詳細步驟。

[1]. (14%) Let

$$f(x)=\begin{cases}\dfrac{\sin\left(x^2\right)}{x^2}, & \text{for } x\neq 0,\\ 1, & \text{for } x=0.\end{cases}$$

(a) Is $f(x)$ continuous at $x=0$? Why?

(b) Is $f(x)$ differentiable at $x=0$? Why?

[2]. (14%)

(a) Let $f(x)=3^x\sin\left(\dfrac{3\pi}{4}x\right)$. Find the equation of the tangent line at $x=2$.

(b) Let $f(x,y)=2e^{-(x+1)^2-(y-2)^2}$. Find the equation of the tangent plane at $(-1,1)$.

[3]. (16%) Let $a,b,c>0$. Use any method in calculus to show that

(a) the area of an ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is πab.

(b) the volume of an ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$ is $\frac{4}{3}\pi abc$.

[4]. (12%) Determine the interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^{n+1}}$.

[5]. (20%) Evaluate the following integrals.

$$(a)\quad \int_2^{\infty}(x-2)^2e^{-x}dx \qquad (b)\quad \int_{-1}^{0}\frac{x^3-1}{x^2-2x+1}dx$$

[6]. (12%) Solve the differential equation $y'+2y=e^{3x}$ with $y(0)=1$.

[7]. (12%) Show that the Laplace equation $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}=0$ in the coordinates (r,θ), where $x=r\cos\theta$ and $y=r\sin\theta$, is

$$\frac{\partial^2 f}{\partial r^2}+\frac{1}{r}\frac{\partial f}{\partial r}+\frac{1}{r^2}\frac{\partial^2 f}{\partial\theta^2}=0.$$

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