# 國立高雄大學一百學年度研究所碩士班招生考試試題

科目：工程數學
考試時間：100 分鐘

系所：
電機工程學系(通訊組)
本科原始成績：100 分

是否使用計算機：是

下列線性代數考試題目共一大題、六小題。請依序進行作答。

1. Let $\mathbf{A}=\begin{bmatrix} 2 & 1 & -2 \\ 1 & 1 & 1 \\ -2 & 1 & 2 \end{bmatrix}$.
   a. Find the determinant of the matrix $\mathbf{A}$? (4%)
   b. Find the rank of the matrix $\mathbf{A}$? (4%)
   c. Find the eigenvalues of the matrix $\mathbf{A}$? (12%)
   d. Suppose that the three eigenvectors are normalized to become $\vec{u}=[u_1, u_2, u_3]^T$, $\vec{v}=[v_1, v_2, v_3]^T$, and $\vec{w}=[w_1, w_2, w_3]^T$, respectively, under the constraint that $u_1 \geq v_1 \geq w_1 \geq 0$. Find $\vec{u}$, $\vec{v}$, and $\vec{w}$? (15%)
   e. Suppose that the angle between $\vec{u}$ and $\vec{v}$ is $\theta_1$, the angle between $\vec{v}$ and $\vec{w}$ is $\theta_2$, and the angle between $\vec{w}$ and $\vec{u}$ is $\theta_3$, respectively. Find $\sum_{i=1}^{3}\cot^2\theta_i=$? (5%)
   f. Find the matrix of $\mathbf{A}^4$? (10%)

下列機率考試題目共二題。請依序進行作答。

2. Suppose that for some distribution with the random variable $X$, the probability density function can be shown as $f_X(x)=2x\cdot e^{-x^2}$, for $x>0$; and $f_X(x)=0$ for $x\leq 0$. Given that there is another random variable $Y$, with the condition $Y=X^2$.
   a. Find the expected value of $X$. (10%)
   b. Find the mode of $X$. (10%)
   c. Find the probability density function of $Y$, $f_Y(y)$. (10%)

3. Let $Z$ be the discrete random variable of the number of breakdown for some system per year. Suppose that during the 3-year period, the probability of two breakdowns is half the value in comparison with the probability of three breakdowns.
   a. Find the mean time of breakdown for such a system. (10%)
   b. Find the probability that at least two breakdowns occur in the second half of this year, from Jul. 1 to Dec. 31. (10%)