科目：微積分
考試時間：100 分鐘

系所：
統計學研究所(風險管理組)　　是否使用計算機：否
本科原始成績：100 分

1. Evaluate the following integrals:

   (a) (10%) $\int_0^\infty x^{20} \exp\{-3x\}dx$

   (b) (10%) $\int_0^\infty x^4 \exp\{-x^2\}dx$

   (c) (10%) $\int_0^{\pi/8} (\sin x \cos^3 x)/(2\cos^2 x - 1)dx$

2. Evaluate the following limits (if it exists):

   (a) (10%) $\lim_{n\to\infty} (1 + \frac{1}{n^2-1})^{2n^2+1}$

   (b) (10%) $\lim_{n\to\infty} (x + \exp\{2x\})^{1/x}$

3. Determine whether the following sequences converges or diverges:

   (a) (10%) $(n^2 + 2n - 2)/(3n^2)$

   (b) (10%) $f(n) = n^{2/3} \sin(n!)/(n+1)$

4. (15%) Let $f(x) = x - x^2, g(x) = ax$. Determine a so that the region above the graph of g and below the graph of f has area $9/2$.

5. (15%) Show that the equation $x^2 = x\sin x + \cos x$ has exactly two real roots of x.