科目：基礎數學
考試時間：100 分鐘

系所：
統計學研究所(統計組)
本科原始成績：100 分

是否使用計算機：否

1. Evaluate the following integrals:

   (a) (8%) $\int_0^\infty x^5 \exp\{-x^3\}dx$

   (b) (10%) $\int_0^{\pi/2} \frac{\sin x + \sin 2x}{1+\cos^2 x}dx$

2. Evaluate the following limits (if it exists):

   (a) (10%) $\lim\limits_{n\to\infty} (n^{-3} + n^{-2}) \sum_{k=1}^n (k^2 + k)$

   (b) (8%) $\lim\limits_{x\to 0} \frac{\sin 5x}{\sin x}$

3. (12%) A function f is continuous everywhere and satisfies the equation

$$\int_0^x f(t)dt = -\frac{1}{2} + x^2 + x\sin 2x + \frac{1}{2}\cos 2x$$

   for all real x. Compute $f(\pi/4)$ and $f'(\pi/4)$.

4. (12%) Given that the derivative $f'(a)$ exists. Derive $\lim\limits_{h\to 0}[f(a+2h)-f(a+h)]/2h$ in term of $f'(a)$

5. (20%) If A and $I + A$ are $k \times k$ nonsingular matrices, show that

   (a) $A^{-1} + I$ is nonsingular

   (b) $(A + I)^{-1} + (A^{-1} + I)^{-1} = I$

6. (20%) Let S be spanned by $x_1^T = [1,1,0,-1]$ and $x_2^T = [0,1,1,1]$; the vector y is defined by $y^T = [1,2,-1,1]$. Find vector z_1 and z_2 such that $y = z_1 + z_2$ where z_1 is in S and z_2 is in $S^\perp$.